Accounting for Needs when Sharing Costs[*]

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Abstract

We introduce needs in the rate-setting problem for essential services, like water or electricity. The goal is to ensure that households with higher needs are not penalized, all the while holding them responsible for their consumption. We show that conventional methods like monetary subsidies cannot achieve this goal in a budget-balanced way. Instead, we characterize axiomatically two families of cost-sharing rules, each favoring one aspect—compensation or responsibility—over the other. A focal solution, dubbed the *utility-free solution*, emerges as a desirable compromise when households differ only in their needs. We identify specific variants of these rules that protect small consumers from the cost externality imposed by larger consumers. Lastly, we show how one can implement these schemes with realistic informational assumptions; i.e., without making explicit interpersonal comparisons of needs and consumption.

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1 Introduction

Some public utilities, like water and wastewater services, or electricity, are essential to achieving a decent standard of living. The European energy crisis of 2022 is a manifestation of the affordability issue surrounding the need to consume energy. Similarly, rolling blackouts and clean water shortages in various parts of the U.S. due to decaying or otherwise inadequate infrastructure point to the problem of sustainability and, in turn, to the issue of insufficient financing.

In a society where households differ in terms of their needs for essential services, these services should be financed sustainably while taking needs into account when setting rates. In practice, commendable efforts have been made in this regard, with rate schedules typically taking the form of multi-part tariffs (block pricing), including discounts given to households with higher needs (for the case of water supply in the US, see AWWA, 2017). These discounts can take the form of rebates to low-income households, which may be subsidized by a higher overall rate structure. Alternatively, increasing-block rate schedules subsidize the lowest block through rate premiums for large consumers, hence affording all households a low rate to meet basic needs. In the case of water services, this also addresses the issue of resource conservation.[1] Nevertheless, while these practices recognize the fact that some households should be subsidized, the design of such subsidies, both in shape and in magnitude, is largely left to rule-of-thumb considerations.[2]

Our aim is to design budget-balanced pricing schemes that do not unduly penalize consumers for having higher needs. As we shall see, attempting to fully shield consumers from an increase in their own needs, while sharing fairly the burden among others, is impossible (Proposition 1). Some conventional solutions like needs-based rate schedules or monetary subsidies take slightly different approaches, yet cannot realistically achieve these goals. We explore in formal detail in Section 4 why this is the case.

Our fairness objective is distinct from the conventional notion of "affordability". Instead, we require that households with higher needs—e.g., because they are of larger

[1] The recent move towards "water budget-based rates" or, more accurately, to "sustainable" rate design in some U.S. municipalities reflects these concerns (Barr and Ash, 2015; Barraqué and Montginoul, 2015; Dinar and Ash, 2015)

[2] For example, the M1 Manual of the American Water Works Association, a highly-regarded reference by North American water utilities, gives surprisingly little guidance on how to determine rate blocks: "Generally, rate blocks should be set at logical break points." (AWWA, 2022, p.107)

size—are not at a disadvantage in their ability to achieve a given welfare level. Hence, while affordability is generally a measure of financial pressure exerted on lower-income households, our fairness criterion applies to the entire population. We elaborate on the distinction between affordability and our fairness objective in Section 2.

We develop a framework to formally take matters of partial responsibility into account when devising rates for utility services, which we will assume to be water or electricity services to fix ideas. Each consumer is summarized by their consumption and their needs, which may differ from one consumer to the next. While we take the view that consumers should not be penalized for their needs, we deem them fully responsible for their consumption beyond those needs.

Our approach builds on the axiomatic framework of liberal egalitarianism, which aims at compensating differences in "non-responsibility" characteristics while rewarding differences in characteristics under one's control. Classically, in a labor setting, individuals are deemed responsible for their effort but have no control over their talents. Here, consumers have no control over their needs but are responsible for their consumption beyond that amount.

To be clear, our view of needs does not necessarily coincide with what are customarily labeled "basic needs"—say, 50 liters of clean water per person per day (Gleick, 1996)—but should be understood more broadly. In particular, the notion of needs may not only be physiological, but can vary geographically, culturally, as well as individually. We adopt the view that *one's needs is a consumption level that they should not be held responsible for consuming*. Whatever the underlying interpretation, it turns out that consumption is thus a "hybrid" characteristic of sorts: the portion required to meet one's needs falls into the non-responsibility category, whereas the remainder falls into the sphere of responsibility. In the context of water services, this may lead to subsidies at low consumption levels—required for food and hygiene—but not for discretionary uses—like filling a swimming pool.

A general theme of that literature is that the two desiderata of compensation and responsibility are incompatible (Bossert, 1995; Bossert and Fleurbaey, 1996; Cappelen and Tungodden, 2006a). Accordingly, one must set less ambitious goals for redistributive policies. This is typically done by giving priority to one ideal—compensation or responsibility—while limiting the scope of the other (Fleurbaey, 2008, and references therein), leading to the *Egalitarian Equivalent* and *Conditional Equality* solutions, respectively. Likewise, we characterize two polar families of solutions: *Egalitarian*

Equivalent solutions emphasize compensation for differences in needs (Theorem 1) while *Conditional Equality* solutions emphasize responsibility for excessive consumption (Theorem 2).

Contrasting with previous results, the solutions we obtain are not unique, but are instead families of solutions, because they depend on two additional dimensions that the literature is currently not equipped to handle: how to account for "hybrid" characteristics and how to account for cost externalities. The latter is embodied by the nonlinearity of the cost function, which links the consumers through the requirement of balancing the budget. Regarding the former, each family of solutions will produce different solutions depending on how one measures responsibility. Measuring responsibility in terms of consumption (q) beyond needs ($\bar{q}$), formally $r = q - \bar{q}$, or in terms of its fraction relative to one's own needs, $r = (q - \bar{q})/\bar{q}$, are but two examples. We call these views *absolute responsibility* and *relative responsibility*, respectively.

When welfare can be evaluated by means of a (common) utility function, u,—i.e., when consumers may differ in their needs but not in their preferences—and when the planner chooses a responsibility view that matters for the actual well-being of consumers—so that $u = v \circ r$ for some increasing function v—, there exists a unique solution that is actually compatible with a much stronger compensation requirement than when responsibility is computed arbitrarily. We coin this solution the *utility-free rate-function* (Theorem 3). This implies that, when differences in needs summarize the relevant differences across consumers, sufficient knowledge of the (common) preferences can afford greater compatibility between the desiderata of compensation and reward, a sharp contrast with existing results in the literature on liberal egalitarianism.

Even with a specific view on responsibility, much freedom remains regarding how to account for cost externalities within each family of solutions. Indeed, the partial responsibility approach determines what portion of the total cost is devoted to meeting one's needs. How to split the remainder—for which consumers are deemed responsible—falls into the realm of cost-sharing theory. In principle, any cost-sharing rule can be associated with any family of solutions and any responsibility view.

Given the nature of the service at hand, we propose the well-known serial cost-sharing rule (Moulin and Shenker, 1992) because it exhibits strong fairness properties both when costs are convex and concave (Moulin, 1996). This leads us to characterizing *needs-adjusted serial solutions* (Propositions 5-7), which are the counterparts to the general solutions of Theorems 1-3 when requiring the additional property of *Independence of Higher Responsibility*, the requirement that my bill not depend on consumers who bear more responsibility than me. Section 6 displays how to construct the needs-adjusted serial solutions.

Lastly, in Section 7, we show how one can implement the above schemes with realistic informational assumptions; i.e., without making explicit interpersonal comparisons of needs or consumption, which would prove very difficult and possibly counterproductive for all but very small populations. To do so, we use household size as a proxy for needs and denote by $\bar{q}_s$ the needs of a household of size s. Using aggregate information to summarize distributional aspects, we design rate schedules that otherwise explicitly depend on the sole individual characteristics of households.

Although our results apply to all cost functions, an illustrative example can be useful. Consider affine costs of the form $C(Q) = F + cQ$, with $F, c > 0$, where Q is the aggregate demand of the population.[3] When taking the absolute responsibility view, $r = q - \bar{q}_s$, the *serial utility-free rate function* yields the following bill for households of size s that consume q:

$$x^{abs}(q, s) = \frac{F + c\bar{Q}}{N} + c\left(q - \bar{q}_s\right), \tag{1}$$

where $\bar{Q}$ is the quantity needed to cover the needs of the entire population, and N is the total number of households. In addition to splitting the fixed cost equally, this rate schedule shares the cost of the population's needs equally before pricing consumption at marginal cost (minus a rebate equal to the cost of meeting one's own needs). Note that, by pricing at marginal cost, this rate function implements the efficient consumption level: accounting for needs is compatible with economic efficiency.

The rate schedule changes significantly under the relative responsibility view, $r = \left(q - \bar{q}_s\right)/\bar{q}_s$. Assuming responsibility is identically distributed across types, we obtain the following bill for households of size s that consume q:

$$x^{rel}(q, s) = \frac{F}{N} + \frac{c}{\bar{q}_s / \left(\bar{Q}/N\right)} q \tag{2}$$

[3] Such a cost structure is typical of water and electricity services, which exhibit high fixed costs (infrastructure) and low marginal costs (electricity for pumping and chemicals for treatment). Section 7 also displays similar examples with decreasing returns to scale (quadratic costs).

The result is still a two-part tariff but one where only the fixed cost is split equally. No rebate is granted, and consumption is priced at a rate that is inversely proportional to one's needs. Notice that consumption is no longer priced at marginal cost, which undermines efficiency. Contrasting with the previous example, one concludes that the efficiency of the rate schedule depends choice of the responsibility view.

The fact that (1) and (2) are two-part tariff formulae spells good news for their implementation because municipalities and and utilities routinely work with two-part tariffs. Our approach anchors the breakdown between the fixed portion and the variable rate on the ethical grounds of needs while guaranteeing a balanced budget. Many examples exist where municipalities have difficulty finding the right balance between the fixed and variable components, resulting in budget deficit (Canada's Ecofiscal Commission, 2017).

2 Related Literature

Liberal egalitarianism. Our work expands the literature on liberal egalitarianism in two ways. First, we extend the theory to settings with externalities. To our knowledge, the only other effort in this direction is Billette de Villemeur and Leroux (2011), which tackles the issue of global climate change and the design of transfer schemes between countries to account for their responsibility in current emissions and, possibly, their non-responsibility in past emissions. Externalities are introduced through a (nonlinear) damage function, but needs are absent from their setting.

Our second contribution has to do with our consideration of a characteristic—here, consumption—for which one is both partly responsible and partly non-responsible. Early works in the economics literature on liberal egalitarianism (Bossert, 1995; Bossert and Fleurbaey, 1996; Fleurbaey and Maniquet, 2006; Cappelen and Tungodden, 2006a) considered characteristics to be either responsibility characteristics, like effort, or non-responsibility characteristics, like talent. Cappelen and Tungodden (2006b) were the first to investigate the implications of the choice of "responsibility cut"; remarkably, increasing the realm of non-responsibility characteristics did not necessarily lead to more redistribution.

Since then, several works have considered various interpretations of the responsibility cut. For example, Billette de Villemeur and Leroux (2011) have considered varying degrees of historical responsibility in past GHG emissions. Closer to the work at hand, Ooghe and Peichl (2014) and Ooghe (2015) introduced the notion of "partial control" over some characteristics to handle different degrees of responsibility in any given characteristic. According to this "soft cut", an consumer may be responsible for, say, only 30% of their intellectual skills, the remainder being attributable to inborn abilities or environmental factors. Our view of consumption as a hybrid characteristic differs from theirs in that we deem households fully non-responsible for the portion aimed at satisfying their needs, but fully responsible for the remaining portion, viewed as discretionary. In other words, the responsibility cut we consider is quantitative, where the responsibility in consumption depends on the value of consumption, rather than qualitative, where the extent of responsibility in a characteristic is independent of that characteristic's value.

Fair division. Despite mounting empirical evidence suggesting that needs are a relevant ingredient of fairness, the literature on fair division has only recently considered needs in a formal fashion. Specifically, although in a setting different from ours, Bergantiños et al. (2012) and Manjunath (2012) modify the classical rationing problem—where a fixed social endowment must be divided among several recipients—to account for a minimal requirement. There, consumers are indifferent between receiving less than this minimal share and receiving nothing.

Because we ask for full cost recovery, the relevant strand of the fair division literature is that of cost sharing. Yet, this literature does not explicitly address the issue of needs. The closest work in that direction leads to a sharing rule that protect small consumers when costs are convex (Moulin and Shenker, 1992): the serial cost sharing rule. We build upon this sharing rule to complement our approach in Section 6.

Needs and affordability. Although ample empirical evidence indicates that needs are a relevant criterion for fair distribution (Konow, 2001, 2003; Nicklisch A, and F. Paetzel, 2020; Bauer et al., 2022), needs are seldom made explicit as a basis for fair compensation in the economics literature.[4] The general consensus seems to be that such a specification is unwarranted because consumers take their needs into account when making purchasing or labor decisions.

Instead, the economics literature has primarily turned to the concept of *affordability* as a metric to evaluate fair distribution (Kessides et al., 2009, and many references

[4]Exceptions include Mayshar and Yitzhaki (1996), Ebert (1997), Trannoy (2003), Duclos et al. (2005), and Fleurbaey et al. (2014).

therein) The aim of affordability is to ensure that all households can reasonably comfortably purchase goods and services that are necessary to achieve a decent standard of living. Interestingly, however, the very concept of affordability rests on the ability to meet one's needs, a feature that is particularly explicit in Canada's Mortgage and Housing Corporation (2019). Indeed, we talk about the affordability of items like housing, food and energy, but never about the affordability of fine jewelry or other luxury goods, for example. So affordability-concerned economists are at least implicitly aware that there exists a such thing as "needs". We suspect the reluctance to spell out the needs of consumers as a separate variable is mainly a practical one—say, trusting that revealed preferences will be more accurate than outside measurement, possibly rightfully so—rather than a conceptual one. Here, we choose to make explicit the needs of consumers because, despite likely measurement errors; we believe that doing so sheds light on how to better set rates for essential services.

Moreover, we claim that the goal of affordability, although valuable in its own right, is but a sufficient condition for the fair pricing of essential services. This is because affordability only focuses on the lower end of the income distribution (vertical equity) but is silent about how to treat mid- to high-income households. As such, affordability cannot prevent the inequity that stems from wealthy households with different needs, which is a matter of horizontal equity: among otherwise identical households, those with higher needs should not end up with fewer opportunities. These two equity dimensions only coincide in the very specific case where needs and income are perfectly (negatively) correlated, which is generally not the case.[5]

Hence, while related, our approach can be seen as orthogonal to that of affordability. We merely aim to ensure that no one is unduly penalized for their needs, which has consequences for the pricing structure. Whether the distribution of additional funds to low-income households is then needed to achieve affordability is a separate issue.

[5]Relatedly, Sallee (2019) finds that, though theoretically possible, achieving a Pareto improvement by recycling revenue from a carbon tax is practically infeasible due to the insufficient correlation between the variables that determine the transfer scheme and those that determine the tax burden. In our setting, there is every reason to suspect that the correlation between income and needs is insufficient to warrant being able to handle differences in needs by simply tackling affordability.

3 Model and notation

The Model. Let $N = \{1, ..., n\}$ be the set of consumers. Consumer i consumes a quantity of energy[6] $q_i \geq 0$. Serving total energy demand, $Q = \sum_{i=1}^{n} q_i$, costs $C(Q) \geq 0$, where C is an increasing cost function.[7] We denote by Γ the class of cost functions.

Full cost recovery is essential to the sustainability of the infrastructure.[8] Thus, we require that the consumers' energy bills, the x_i's, cover the total cost:

$$\sum_{i=1}^{n} x_i = C(Q). \tag{3}$$

Our aim is to define appropriate formulas to compute the energy bills of consumers. We restrict ourselves to the case where no profits are made, owing to the public nature of the service, so that the budget constraint (3) is binding whenever all consumers consume at least their needs ($q_i \geq \bar{q}_i$ for all i).[9]

The needs of consumer i, in terms of energy consumption, are denoted $\bar{q}_i \geq 0$. We adopt a quasi-linear setup, where Consumer i's utility level is defined by:

$$U_i(q_i, \bar{q}_i, x_i) = u_i(q_i, \bar{q}_i) - x_i.$$

The utility function u_i, which is possibly consumer specific, is defined on $\mathbb{R}_+^2$ and is assumed to be increasing in q_i and decreasing in $\bar{q}_i$. When consumers consume exactly their needs, they share a common utility level that, without loss of generality,

[6]The best term would be "utility services" that, in addition to being unwieldy, is unfortunately too close to the term "utility" ubiquitous in consumer theory. Instead, we shall use the term "energy" throughout, but the analysis applies equally well to, say, water services as it does to electricity.

[7]We use the following convention: by 'increasing' we mean 'strictly increasing'. We use the term 'non-decreasing' when the monotonicity is not strict. Similarly, by 'positive' we mean 'strictly positive', and use 'nonnegative' when zero is not excluded.

[8]For example, while it remains an empirical matter whether pricing water actually leads to economic efficiency in practice, it is widely recognized that full cost recovery is essential to the sustainability of the infrastructure (Massarutto, 2007; AWWA, 2017; Canadian Water and Wastewater Association, 2015) and is "a key preoccupation" of many OECD countries (OECD, 2010). Still in the context of water services, Massarutto (2007) identifies three important benefits of recovering costs through the pricing structure: to "ensure the viability of water management systems", to "maintain asset value over time", and to "guarantee the remuneration of inputs".

[9]In practice, however, this is not an issue because none of our solutions will induce consumption below one's needs.

we shall set to zero. Formally,

$$u_i(\bar{q}_i, \bar{q}_i) \equiv 0, \qquad \forall \bar{q}_i \geq 0, \forall i \in N.$$

We shall also assume u_i to be continuously differentiable, with positive cross-derivative—$\partial^2 u_i / \partial q_i \partial \bar{q}_i > 0$—to reflect the fact that higher needs reduce the marginal benefit from consumption. We denote by Υ the class of utility functions.

Defining responsibility. Our aim is to design a pricing rule that does not penalize consumers with higher needs while still taking individual responsibilities with respect to consumption into account. To do so, we must define the consumers' sphere of responsibility. We shall consider that consumers are not responsible for their needs, $\bar{q}_i$, but are responsible for any consumption beyond those needs. The extent of their responsibility can be measured in many different ways. For the sake of generality, we define a real-valued function, $r(q_i, \bar{q}_i)$, defined on $\mathbb{R}^2_+$, which is increasing in energy consumption q_i, decreasing in needs $\bar{q}_i$, and normalized to zero whenever $q_i \leq \bar{q}_i$. We shall further assume that r is differentiable. When no confusion is possible, we abuse notations slightly by denoting $r_i = r(q_i, \bar{q}_i)$. For example, $r(q_i, \bar{q}_i) = q_i - \bar{q}_i$, which we refer to as the *absolute responsibility* view, considers that consumers are only responsible for the amount of discretionary consumption. A more nuanced approach, the *relative responsibility* view, $r(q_i, \bar{q}_i) = (q_i - \bar{q}_i)/\bar{q}_i$, assigns responsibility relative to the consumer's needs (e.g., household size).

A *consumption-needs profile* (or simply *a profile*) is a list of n consumption-needs pairs that we shall denote $(\mathbf{q}, \bar{\mathbf{q}}) \in \mathbb{R}^{2n}_+$, abusing notations slightly.[10]

Having defined the notion of responsibility, we can now share the total cost according to the responsibility profile, $\mathbf{r} \equiv r(\mathbf{q}, \bar{\mathbf{q}}) = (r_1, r_2, ..., r_n) \in \mathbb{R}^n_+$. A *rate function* takes all the information in the economy into account and is a mapping $x : \mathbb{R}^{2n}_+ \times \mathcal{R} \times \Upsilon^n \times \Gamma \to \mathbb{R}^n$ such that $\sum_{i \in N} x_i(\mathbf{q}, \bar{\mathbf{q}}, r, \mathbf{u}, C) = C(Q)$, where $\mathbf{u} = (u_1, ..., u_n)$ is the profile of the consumers' utility functions.

A minimal fairness requirement we shall adopt throughout is that rate functions should satisfy *anonymity*. Formally, we require that, for any permutation of con-

[10] We shall adopt the convention that boldface type refers to the vector of the relevant variables. E.g., $\mathbf{q} = (q_1, ..., q_n)$ and $\bar{\mathbf{q}} = (\bar{q}_1, ..., \bar{q}_n)$.

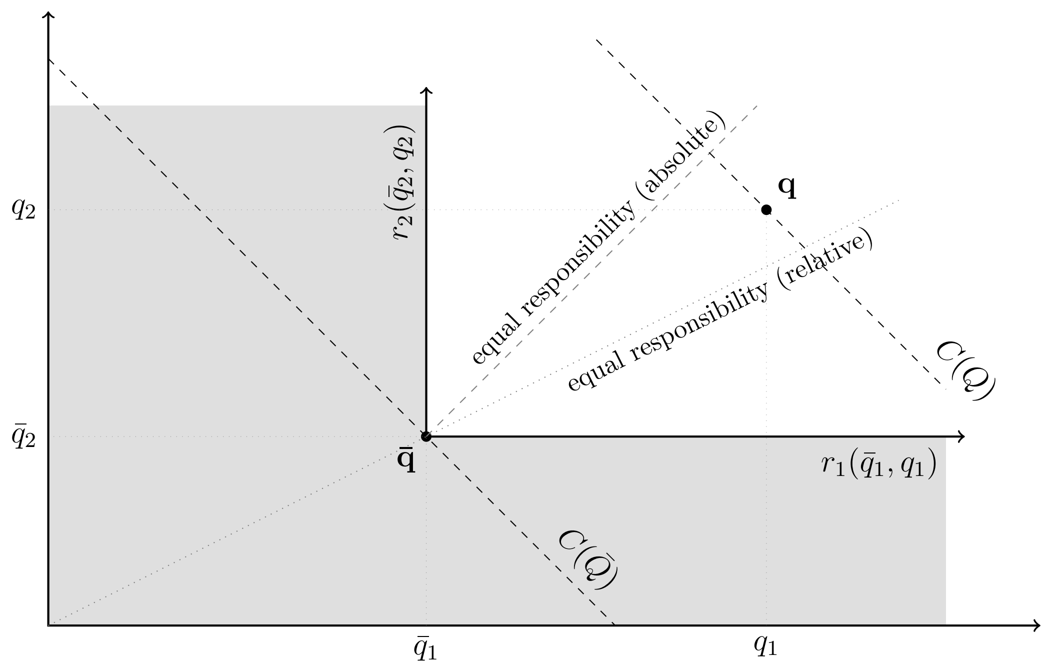

Figure 1: Responsibility is measured from $\bar{\mathbf{q}}$. Given the position of $\mathbf{q}$ relative to $\bar{\mathbf{q}}$ in this figure, if responsibility is defined as $q_i - \bar{q}_i$ (absolute responsibility) consumer 1 is considered to bear more responsibility than consumer 2 in her discretionary consumption. If it is defined as $(q_i - \bar{q}_i)/\bar{q}_i$ (relative responsibility), the reverse holds.

sumers, $\pi : N \to N$:

$$x_{\pi(i)}\left(\mathbf{q}_{\pi}, \bar{\mathbf{q}}_{\pi}\right) = x_i\left(\mathbf{q}, \bar{\mathbf{q}}\right) \qquad \text{for all } i \in N,$$

where $\mathbf{q}_{\pi}$ (resp. $\bar{\mathbf{q}}_{\pi}$) is the vector of consumption (resp. needs) after permutation of consumers along π.

Remark. Anonymity implies the equal treatment of equals: $(q_i, \bar{q}_i) = (q_j, \bar{q}_j) \implies x_i(\mathbf{q}, \bar{\mathbf{q}}) = x_j(\mathbf{q}, \bar{\mathbf{q}})$. Two consumers with identical needs and identical consumption must pay the same bill.

Section 7 will be devoted to obtaining explicit formulas based on illustrative examples. Until then, fix the cost function, C, the utility function profile, $\mathbf{u}$, and the responsibility function, r. As a result, we abuse notations slightly and write $x(\mathbf{q}, \bar{\mathbf{q}})$ instead of the more cumbersome $x(\mathbf{q}, \bar{\mathbf{q}}, r, \mathbf{u}, C)$.

4 Why full needs protection is an unrealistic pursuit

When tasked with treating fairly consumers with different needs, a natural inclination is to operate in two steps. First, address the heterogeneity in needs through some compensation mechanism. Then, once consumers are on an equal footing, apply a classical pricing scheme that ignores differences in needs.

The underlying objective of the compensation stage is to shield consumers with higher needs. Formally, when the needs of consumer i increase from $\bar{q}_i$ to $\bar{q}'_i$, that consumer's well-being should not be affected:

$$u_i\left(q_i, \bar{q}'_i\right) - x_i\left(\mathbf{q}, \bar{\mathbf{q}}'\right) = u_i\left(q_i, \bar{q}_i\right) - x_i\left(\mathbf{q}, \bar{\mathbf{q}}\right), \tag{4}$$

where $\bar{\mathbf{q}}'$ is the vector $\bar{\mathbf{q}}$ whose i'th coordinate has been replaced with $\bar{q}'_i$.

Rearranging yields:

$$u_i\left(q_i, \bar{q}'_i\right) - u_i\left(q_i, \bar{q}_i\right) = x_i\left(\mathbf{q}, \bar{\mathbf{q}}'\right) - x_i\left(\mathbf{q}, \bar{\mathbf{q}}\right), \tag{5}$$

while dividing through by $\bar{q}_i' - \bar{q}_i$ and taking the limit as $\bar{q}_i'$ approaches $\bar{q}_i$ yields:

$$\frac{\partial u_i}{\partial \bar{q}_i} = \frac{\partial x_i}{\partial \bar{q}_i}. \tag{6}$$

Equation (6) states that the shape of the rate schedule should be the same as that of the consumers' utility function in order to fully absorb the utility consequences of an increase in i's needs.

On the other hand, the motivation for shielding Consumer i from the increase in her own needs is that this increase is not her responsibility. But by that reasoning, the increase in Consumer i's is not Consumer j's responsibility either. So Consumer j should not be affected any more than Consumer i.[11] Hence, because Consumer i's well-being is unchanged, Consumer j's well-being must also be unaffected. And because a change in $\bar{q}_i$ only impacts Consumer j's through the rate schedule (but not through the utility function), x_j must be unaffected. Formally,

$$\frac{\partial x_j}{\partial \bar{q}_i} = 0 \tag{7}$$

for all $j \neq i$.

However, the rate schedule must cover the total cost: $\sum_{i=1}^{n} x_i = C(Q)$. Upon considering an infinitesimal change in $\bar{q}_i$, and noticing that this does not affect the total cost, we obtain:

$$\frac{\partial x_i}{\partial \bar{q}_i} + \sum_{j \neq i} \frac{\partial x_j}{\partial \bar{q}_i} = 0. \tag{8}$$

According to (6) and (7), this turns into:

$$\frac{\partial u_i}{\partial \bar{q}_i} = 0. \tag{9}$$

We have thus established the following proposition:

Proposition 1. *A rate structure can fully shield a consumer from an increase in their needs, while not imposing a burden on others, only when needs do not impact that consumer's utility.*

In other words, we obtain the somewhat paradoxical result that the issue of needs

[11]The property that the burden befalls on Consumer i and Consumer j equally plays a key role in the rest of the article, and is denoted Solidarity from Section 5 onward.

can only be addressed in a satisfactory way when needs do not matter. This is tantamount to saying that full needs protection is impossible and should not be pursued.

That being said, we shall argue throughout the remainder of the article (Section 5 onward) that ensuring that consumers are not *unduly* affected by changes in needs is a worthwhile pursuit that deserves careful consideration. But first, we mention two commonplace approaches to handling needs: needs-based rate schedules and monetary subsidies. The first runs into the same impossibility as Proposition 1, but is worth considering to see how unrealistically restrictive it is *even before considering needs protection* (Proposition 2). As for monetary subsidies, they make it such that the net payments need not cover the full cost, so Proposition 1 does not apply. We shall see, however, that needs protection imposes severe restrictions on the utility functions and on the cost function (Propositions 3 and 4).

4.1 Needs-based rate schedules

A simple way of accounting for differences in needs while simultaneously holding consumers responsible for consumption beyond their needs is to make rate schedules needs-contingent. In practice, this can take the form of conditioning the rate schedule on the size of the household, for example. Formally:

Axiom. ***(Equal Rate Schedule for Equal Needs, ERSEN)***
The functions $q_i \mapsto x_i(\mathbf{q}, \bar{\mathbf{q}})$ *and* $q_j \mapsto x_j(\mathbf{q}, \bar{\mathbf{q}})$ *must be identical whenever* $\bar{q}_i = \bar{q}_j$.

In particular, **ERSEN** guarantees that consumers with equal needs face the same rate schedule. As it turns out, however, **ERSEN** is unfeasible unless the cost function is linear:

Proposition 2. *No rate function satisfies* ***ERSEN*** *unless the cost function is linear:* $C(Q) = \lambda Q$ *for some* $\lambda > 0$.

Proof. Let $(\mathbf{q}, \bar{\mathbf{q}}) \in \mathbb{R}_+^{2n}$ such that $\bar{q}_i = \bar{q}_j$ for some $i \neq j$. By budget balance, the rate schedule of consumer i, $f : q_i' \mapsto x_i((q_i', \mathbf{q_{-i}}), \bar{\mathbf{q}})$, writes as follows:

$$f(q_i') - f(q_i) = C(Q - q_i + q_i') - C(Q) \qquad \forall q_i' \in [\bar{q}_i, +\infty). \tag{10}$$

By **ERSEN**, the function f cannot depend on q_j, so that the above expression must also hold if we can replace q_j by q_j':

$$f(q_i') - f(q_i) = C\left(Q - q_i + q_i' - q_j + q_j'\right) - C\left(Q - q_j + q_j'\right), \tag{11}$$

for all $\left(q_i', q_j'\right) \in [\bar{q}_i, +\infty) \times [\bar{q}_j, +\infty)$. Taken together, Expressions (10) and (11) yield:

$$C(Q - q_i + q_i') - C(Q) = C\left(Q - q_i + q_i' - q_j + q_j'\right) - C\left(Q - q_j + q_j'\right), \tag{12}$$

for all $\left(q_i', q_j'\right) \in [\bar{q}_i, +\infty) \times [\bar{q}_j, +\infty)$.

Already, Expression (12) suggests that C increases at a constant rate. We prove this formally by rewriting the expression as a functional equation. Let $h > 0$ and consider $q_i' = q_i + h$ and $q_j' = q_j + h$. Expression (12) becomes:

$$C(Q + h) - C(Q) = C(Q + 2h) - C(Q + h) \qquad \forall h \geq 0. \tag{13}$$

Rearranging and defining $g : h \mapsto C(Q + h)$ on $\mathbb{R}_+$ yields:

$$g(2h) + g(0) = 2g(h) \qquad \forall h \geq 0. \tag{14}$$

Expression (14) must hold for all h and thus defines a functional equation in g. This is a well-known Cauchy equation (Aczél, 1967), which requires g—and therefore C—to be linear in its argument. Having started from an arbitrary profile $(\mathbf{q}, \bar{\mathbf{q}})$, linearity follows on the full domain of C. □

ERSEN effectively requires that the rate schedule an consumer faces depends only on the profile of needs, but not on the consumption vector. However, this ignores the interdependence that exists between consumers through the cost function should the latter be non-linear. Proposition 2 expresses the fact that, should consumers indeed be interdependent through the cost function, rate schedules cannot be determined *ex ante* on the sole basis of needs.[12]

Naturally, if the cost function C is linear, of the form $C(Q) = cQ$ for some $c > 0$, the cost-sharing problem becomes substantially simpler: **ERSEN** requires that each consumer pay $x_i(\mathbf{q}, \bar{\mathbf{q}}) = y_i(\bar{\mathbf{q}}) + cq_i$ where the functions $(y_i)_{i \in N}$ are symmetric—$\bar{q}_i =$

[12]For a general proof of the incompatibility between budget balance and equal treatment of equals, albeit where needs are absent, see Billette de Villemeur and Leroux (2016).

$\bar{q}_j \implies y_i(\bar{\mathbf{q}}) = y_j(\bar{\mathbf{q}})$—and sum up to zero, $\sum_{i \in N} y_i \equiv 0$.

For example, any linear function of the difference between one's need and the average need of the population will work: $y_i(\bar{\mathbf{q}}) = \lambda(\mu - \bar{q}_i)$, for some $\lambda \geq 0$, where $\mu = \sum_j \bar{q}_j$. This yields,

$$x_i(\mathbf{q}, \bar{\mathbf{q}}) = \lambda(\mu - \bar{q}_i) + cq_i, \tag{15}$$

for some $\lambda \geq 0$.

Other solutions exist, like those that consider coarse needs categories. For example, fix $\bar{q}^*$ and $\bar{q}^{**}$ to be, respectively, a relatively low level of needs (e.g., the needs of a 2-person household) and a realtively high level of needs (e.g., the needs of a 5-person household). Denote by l (resp. h) the number of households whose needs are below $\bar{q}^*$ (resp. above $\bar{q}^{**}$). Subsidizing high-needs families by taxing low-needs households is a viable solution:

$$x_i(\mathbf{q}, \bar{\mathbf{q}}) = cq_i + \begin{cases} \lambda h & \text{if } \bar{q}_i\text{¡}\bar{q}^* \\ 0 & \text{if } \bar{q}_i \in [\bar{q}^*, \bar{q}^{**}] \\ -\lambda l & \text{if } \bar{q}_i\text{¿}\bar{q}^{**} \end{cases} \tag{16}$$

While attractively simple, recall that the two-part tariffs in (15) and (16) are only satisfactory if the cost function is linear. That said, if the objective is to share the burden of a change in one consumer's needs across all individuals, one easily checks that the functions $(y_i)_{i \in N}$ must be constants. And, because of the symmetry requirement—$\bar{q}_i = \bar{q}_j \implies y_i(\bar{\mathbf{q}}) = y_j(\bar{\mathbf{q}})$—they must actually all equal zero. Hence, $x_i(\mathbf{q}, \bar{\mathbf{q}}) = cq_i$ and the needs question is entirely evacuated.

4.2 Monetary subsidy

Proposition 2 established that setting needs-based rate scheduled is too demanding a requirement because actual cost functions are generally not linear.

A more flexible approach is to charge according to consumption, $t_i(\mathbf{q})$, as one would in the absence of needs, and grant a subsidy based on needs $s_i(\bar{\mathbf{q}})$,[13] so that the consumer's net payment is $t_i(\mathbf{q}) - s_i(\bar{\mathbf{q}})$. In principle, this allows the service provider to neatly separate the issue of responsibility from that of needs compensation.

[13]Note that consumer i's rebate cannot depend solely on consumer i's own needs—so that we must have $s_i(\bar{\mathbf{q}})$ rather than $s_i(\bar{q}_i)$—otherwise we run into similar impossibilities as in Section 4.1.

Moreover, because it does not rely on financing the total cost (because of the subsidy) there is hope yet that we can escape the negative result of Proposition 1.

Specifically, the consumption charges should cover the total cost,

$$\sum_i t_i(\mathbf{q}) = C(Q), \tag{17}$$

while the monetary subsidies cover the collective cost of meeting the population's needs, [14]

$$\sum_i s_i(\bar{\mathbf{q}}) = C\left(\bar{Q}\right). \tag{18}$$

Moreover, the purpose of the subsidy is to ensure that one does not pay whenever one consumes only one's needs:

$$q_i = \bar{q}_i \implies t_i(\mathbf{q}) - s_i(\bar{\mathbf{q}}) = 0. \tag{19}$$

Proposition 3. *A service charge, $t_i(\mathbf{q})$, coupled with a monetary subsidy, $s_i(\bar{\mathbf{q}})$, to shield consumers from the impact of increases in needs is only effective if all consumers' utility functions are of the form $u_i(q_i, \bar{q}_i) \equiv v_i(q_i) - v_i(\bar{q}_i)$ for some increasing function v_i, possibly individual-specific.*

Proof. Appendix A.1. □

Remark 1. The functional form $u_i(q_i, \bar{q}_i) = v_i(q_i) - v_i(\bar{q}_i)$ actually violates $\partial^2 u_i / \partial q_i \partial \bar{q}_i > 0$ assumed at the outset. The marginal utility of consumption, calculated at a given level q_i, should be larger when needs have not yet been met—$\bar{q}_i > q_i$—than when they have—$\bar{q}_i < q_i$. Yet, the additive separability of u_i precludes this.

Proposition 4.2 establishes strong restrictions on the utility functions for a monetary subsidy to be able to shield individual consumers from increases in their own needs. Yet, because needs are no one's responsibility, one expects the overall rate schedule to share the burden of increases in the needs *of others* in a balanced way (a property we shall dub Solidarity later from Section 5 onward). The following proposition states the further requirements on the utility functions (and on the cost function) for a monetary subsidy to behave as intended.

[14] The reader will have noticed that the total net payments add up to $C(Q) - C\left(\bar{Q}\right)$ rather than $C(Q)$. This implies that some authority (the government, say) subsidizes the essential service at hand. For example, it corresponds to the practical situation where subsidies are granted in the form of welfare checks to households.

Proposition 4. *i) A service-charge-and-subsidy rate schedule that shields consumers from changes in their own needs (as in Proposition 3) can only share the burden equally across all consumers if the cost function is affine—*$C(Q) \equiv cQ + k$*—and if all consumers have utility functions that are linear with the same rate as the cost function:* $u_i(q_i, \bar{q}_i) \equiv c(q_i - \bar{q}_i)$.

Proof. Appendix A.2. □

Proposition 4 further narrows down the set of situations where a service-charge-and-subsidy rate schedule might be satisfactory. Specifically, the cost function must be affine: $C(Q) = cQ + k$. While not as stringent as the requirement of a linear cost function of Proposition 2—$C(Q) = cQ$—it remains very restrictive. In addition, the utility of all consumers must be proportional to the costs: $u_i(q_i, \bar{q}_i) = c(q_i - \bar{q}_i)$. This is conceptually far more problematic than simply applying the rate schedule to the correct cost-sharing situation: it requires a (possibly unlikely) coincidence of material constraints—through the cost function—with subjective appreciations (through preferences).

5 Compensation and Responsibility

The negative results of the previous section are indicative of the strong tension between compensating for one's needs, on the one hand, and holding consumers responsible for their consumption, on the other. In particular, Proposition 2 clearly demonstrates that one must depart from the simplistic view according to which consumers can ignore the impact they have on others, as is assumed to be the case under perfect competition, for instance.

We therefore adopt a more comprehensive view in which bills depend explicitly on the entire profile of consumption and needs. We shall thus stick to our encompassing approach, which aims at financing the total cost, $C(Q)$, by accounting jointly for the q_i's and the $\bar{q}_i$'s. Also, we allow ourselves to consider individual well-being, $u_i(q_i, \bar{q}_i) - x_i$, as a relevant metric for compensation.

In our view, the hallmark of compensation consists in ensuring that differences in needs do not drive differences in well-being. Formally, this amounts to spreading the impacts of a change in one consumer's needs equally across the population:

Axiom. ***(Solidarity)***
For any $i \in N$ *and any two profiles* $(\mathbf{q}, \bar{\mathbf{q}})$ *and* $(\mathbf{q}, \bar{\mathbf{q}}')$ *such that* $\bar{q}'_i \neq \bar{q}_i$ *and* $\bar{q}'_j = \bar{q}_j$ *for all* $j \in N \setminus \{i\}$, *then*

$$\left[u_j\left(q_j, \bar{q}'_j\right) - x'_j\right] - \left[u_j\left(q_j, \bar{q}_j\right) - x_j\right] = \left[u_i\left(q_i, \bar{q}'_i\right) - x'_i\right] - \left[u_i\left(q_i, \bar{q}_i\right) - x_i\right],$$

for all $j \in N$, *where* $x = x(\mathbf{q}, \bar{\mathbf{q}})$ *and* $x' = x(\mathbf{q}, \bar{\mathbf{q}}')$.

Regarding responsibility, a natural requirement is that the portion of costs resulting from consumption above and beyond the needs of the population, $C(Q) - C(\bar{Q})$, should be distributed to consumers according to their responsibility in the total cost. We do this by introducing a *cost-sharing rule*, ξ, to split $C(Q) - C(\bar{Q})$ according to the responsibility profile, $\mathbf{r}$. Keeping with the desideratum of anonymity, we shall consider only symmetric cost-sharing rules:

$$\xi\left(\mathbf{r}, C - C(\bar{Q})\right) \text{ is a symmetric function of the variables } r_i,\ i \in N.$$

The function ξ embodies how we want to hold consumers accountable for their consumption.[15] We shall take ξ as given for now, so as to focus on the articulation between responsibility and compensation. Later, in Section 6, we shall pin down a functional form for ξ with additional desirable properties.

Axiom. ***(Shared Responsibility)***

$$x_k(\mathbf{q}, \bar{\mathbf{q}}) - x_k(\bar{\mathbf{q}}, \bar{\mathbf{q}}) = \xi_k(\mathbf{r}, C - C(\bar{Q})) \qquad \forall k \in N$$

Remark. Our setting encompasses the classical cost-sharing framework when needs are absent (e.g., Moulin, 2002). There, Solidarity is a moot point, and responsibility amounts to compensation ($r_i \equiv q_i$).

As we shall see, the tension between compensation and responsibility remains despite this broader approach (Corollary 1), but becomes manageable provided one chooses to focus more on compensation at the expense of responsibility (Theorem 1) or vice versa (Theorem 2). We will also establish that one can strike a balanced compromise between compensation and responsibility, provided the planner's view on responsibility is one that reflects the preferences of the consumers (Theorem 3).

[15] If needs were not an issue, we would be back to the classical cost-sharing framework where $\xi(\mathbf{q}, C)$ alone defines the shares to be paid (see Moulin, 2002, for a thorough survey).

5.1 Putting compensation first

Turning to compensation first and foremost, we investigate to what extent **Solidarity** is compatible with **Shared Responsibility**. Specifically, we specify how much one must weaken the responsibility requirement in order to maintain an uncompromising view of compensation.

A less demanding requirement than **Shared Responsibility** is to share costs according to ξ only when the needs of all are identical and equal to a reference level, $\bar{q}_0 \in \mathbb{R}_+$:

Axiom. ***(Shared Responsibility for Reference Needs, SRRN)***
For some reference level of needs, $\bar{q}_0 \in \mathbb{R}_+$:

$$[\bar{q}_i = \bar{q}_0, \forall i \in N] \implies [x_k(\mathbf{q}, \bar{\mathbf{q}}_0) - x_k(\bar{\mathbf{q}}_0, \bar{\mathbf{q}}_0) = \xi_k(\mathbf{r_0}, C - C(n\bar{q}_0)), \forall k \in N]$$

where $\bar{\mathbf{q}}_\mathbf{0} = (\bar{q}_0, \bar{q}_0, ..., \bar{q}_0)$ and $r_{0,i} = r(q_i, \bar{q}_0)$ for all $i \in N$.

Clearly, **SRRN** only applies to a small subset of situations and is therefore much less demanding than **Shared Responsibility**. However, it is a necessary weakening to allow **Solidarity** to operate in full force. Indeed, **Solidarity** together with **SRRN** fully determine a family of rate functions, which we call the *Egalitarian Equivalent solutions*,[16] that is parametrized by the chosen reference level of needs, $\bar{q}_0$:

Theorem 1. *A rate function x satisfies **Solidarity** and **SRRN** if and only if $x = x^{EE}$ where, for a given reference level of needs, $\bar{q}_0 > 0$,*

$$\begin{aligned} x_i^{EE}(\mathbf{q}, \bar{\mathbf{q}}) &= \frac{C(n\bar{q}_0)}{n} + \xi_i(\mathbf{r}_0, C - C(n\bar{q}_0)) \\ &\quad + [u_i(q_i, \bar{q}_i) - u_i(q_i, \bar{q}_0)] - \frac{1}{n}\sum_{k=1}^{n}[u_k(q_k, \bar{q}_k) - u_k(q_k, \bar{q}_0)], \end{aligned}$$

for all $i \in N$, where $\mathbf{r}_0 = (r(q_1, \bar{q}_0), r(q_2, \bar{q}_0), ..., r(q_n, \bar{q}_0))$.

Proof. In Appendix A.3. □

x^{EE} measures responsibility relative to the common reference level of needs, $\bar{q}_0$: $r_{0,i} = r(q_i, \bar{q}_0)$ and splits costs accordingly. Differences between actual needs and the reference level are compensated for so as to preserve the relative welfare distribution.

[16] The name reflects the fact that this family of solutions is reminiscent of the egalitarian equivalent allocations in the seminal contribution by Pazner and Schmeidler (1978).

Remark 2. The cost-sharing portion of the transfer, $(1/n)\, C(n\bar{q}_0)+\xi_i\left(\mathbf{r}_0, C - C\left(n\bar{q}_0\right)\right)$, is driven by the consumption profile of the consumers and by the cost structure, but is actually independent of individual needs. By contrast, the redistributive component of the bill, $\left[u_i\left(q_i, \bar{q}_i\right) - u_i\left(q_i, \bar{q}_0\right)\right] - (1/n)\sum_{k=1}^{n}\left[u_k\left(q_k, \bar{q}_k\right) - u_k\left(q_k, \bar{q}_0\right)\right]$, is based on the benefits the consumers derive from consumption and is independent of costs.

The characterization is tight, in the sense that the Egalitarian Equivalent solution does not satisfy a stronger version of responsibility where the cost-sharing rule ξ applies whenever all have the same needs (though not necessarily equal the reference level of needs). This can be shown by considering a profile $(\mathbf{q}, (\bar{q}_1, \bar{q}_1, ..., \bar{q}_1)) \in \mathbb{R}_{+}^{2n}$ such that $\bar{q}_1 \neq \bar{q}_0$ to verify that **SRRN** cannot be satisfied simultaneously using $\bar{q}_0$ and $\bar{q}_1$. The formal proof can be found in Appendix A.4. In particular, this establishes formally that **Solidarity** and **Shared Responsibility** are incompatible:

Corollary 1. *No rate function satisfies* ***Solidarity*** *and* ***Shared Responsibility****.*

5.2 Putting responsibility first

Next, and somewhat symmetrically to the previous section, we assume a strong stance on responsibility—meaning that we insist on an uncompromising version of **Shared Responsibility**—but explore to what extent **Solidarity** must be altered in order to obtain a feasible solution.

The altering of **Solidarity** we shall opt for is, this time, tied to a reference level of responsibility, $r_0 \geq 0$. It requires all consumers to have the same level of well-being if all have a common level of responsibility that is equal to the reference level:

Axiom. ***(Uniform Well-being for Reference Responsibility, UWRR)***
For some reference responsibility level, $r_0 \in \mathbb{R}_+$ *:*

$$\left[r\left(q_i, \bar{q}_i\right) = r_0, \forall i \in N\right] \implies \left[u_i\left(q_i, \bar{q}_i\right) - x_i = u_j\left(q_j, \bar{q}_j\right) - x_j, \forall i, j \in N\right]$$

We find that **Shared Responsibility** and **UWRR** jointly characterize a family of rate functions, which we call *Conditional Equality solutions*,[17] that is parametrized by the choice of a reference responsibility level, r_0:

[17] The name reflects the fact that this family of solutions is reminiscent of the conditional equality solution in Fleurbaey (1995) in a different context.

Theorem 2. *A rate function x satisfies **Shared Responsibility** and **UWRR** if and only if $x = x^{CE}$ where, for some reference level $r_0 > 0$,*

$$x_i^{CE}(\mathbf{q}, \bar{\mathbf{q}}) = \frac{C(\bar{Q})}{n} + \xi_i\left(\mathbf{r}, C - C(\bar{Q})\right) + u_i\left(q_i^0, \bar{q}_i\right) - \frac{1}{n}\sum_{j \in N} u_j\left(q_j^0, \bar{q}_j\right), \qquad (20)$$

for all $i \in N$, where q_i^0 is defined by $r(q_i^0, \bar{q}_i) = r_0$.

Proof. In Appendix A.5. □

Remark 3. The fact that the Conditional Equality solutions satisfy weaker compensation axioms does not mean that the Egalitarian Equivalent solutions are more redistributive. Indeed, for the latter, the parameter $\bar{q}_0$ dictates both the portion of the cost to be shared in an egalitarian fashion and how differences in needs are accounted for. In particular, when $\bar{q}_0 = 0$, the portion of costs to be split equally under x^{EE} is nil—$C(n\bar{q}_0)/n = 0$—and consumers are held responsible for their whole consumption. By contrast, x^{CE} always splits equally the portion of costs corresponding to the needs of the population: $C(\bar{Q})$.

5.3 Choosing the "right" responsibility measure affords a desirable compromise

Theorem 2 is tight because x^{CE} generically does not satisfy a stronger version of the compensation axiom whereby all obtain the same well-being whenever all have the same responsibility (but not necessarily the reference responsibility level, see Lemma 1 in Appendix A.6). The only exception is when the consumers differ only in their needs and the responsibility function, r, reflects the utility derived by the consumers. In that case there exists a rate function that satisfies an even stronger version of the compensation axiom, which requires that when two consumers bear an equal responsibility, their welfare should be equal:

Axiom. ***(Equal Well-being for Equal Responsibility, EWER)***

$$r_i = r_j \quad \Longrightarrow \quad u_i(q_i, \bar{q}_i) - x_i = u_j(q_j, \bar{q}_j) - x_j$$

In fact, when the consumers differ only in their needs and the responsibility function, r, reflects the consumers' utility, **Shared Responsibility** and **EWER** together characterize a unique solution:

Theorem 3. *If $u_i = u \in \Upsilon$, for all $i \in N$ and if $r = \rho \circ u$, for some increasing function $\rho : \mathbb{R} \to \mathbb{R}_+$, the unique rate function satisfying **Shared Responsiblity** and **EWER** is the **utility-free rate function**:*

$$x_i^{UF}(\mathbf{q}, \bar{\mathbf{q}}) = \frac{C\left(\bar{Q}\right)}{n} + \xi_i\left(\mathbf{r}, C - C\left(\bar{Q}\right)\right) \qquad \textit{for all } i \in N.$$

Proof. In Appendix A.6. □

The utility-free rate function, x^{UF}, possesses the pragmatic advantage of not requiring knowledge of the utility function to compute rates. This is a significant advantage over other rate functions when it comes to practical applications. But the normative implications of Theorem 3 are much greater. Importantly, when consumers differ only in their needs, but not in their preferences (so that the theorem applies), cardinal information about preferences is not needed. Indeed, the responsbility ranking, as given by r, coincides with the utility ranking among households (both gross and net of pricing).

Another remarkable feature of the above characterization is that it does not require specifying a reference level of needs or of responsibility. That being said, for Theorem 3 to properly apply, the planner must know the population well enough to choose a responsibility measure that mirrors how consumers actually assess their own well being. This is because choosing $r = \rho \circ u$ amounts to choosing a responsibility measure that actually reflects what matters to consumers: their utility can indeed be written as $u = v \circ r$, with $v \equiv \rho^{-1}$ (see the proof in Appendix A.6 for more details). Nevertheless, should the planner have reasonable grounds to assume identical preferences as a first approximation, the rate function x^{UF}, for a well-chosen responsibility measure, is an attractive candidate to handle both compensation and responsibility.

Finally, note that x^{UF} is a two-part tariff of sorts: it exhibits a fixed fee plus a variable component. Two-part tariffs—like, say, a fixed fee plus marginal cost pricing—face the valid criticism that small users pay a disproportionate amount of the overall cost through the fixed component (Moulin, 1996). In principle, a user who consumes almost nothing faces an effective per-unit price that is almost infinite. By contrast, the fixed component of x^{UF}, $C\left(\bar{Q}\right)/n$, is anchored to the cost of meeting the needs of all. From an individual's perspective, even if a user were to consume

	Shared Responsibility	**SRRN**
Solidarity	None (Corollary 1)	x^{EE} family (Thm 1)
EWER	None, unless $u_i = u \in \Upsilon$, $\forall i$, and $r = \rho \circ u$, then x^{UF} (Thm 3)	x^{EE} family and more
UWRR	x^{CE} family (Thm 2)	x^{EE} family and even more

Table 1: Compatibility table between compensation and responsibility axioms. Each cell indicates the family of rate functions that satisfies both axioms. 'None' stands for an incompatibility between the two axioms.

"only" their needs, and even if those needs were small, meeting one's needs is itself a meaningful consumption level. In turn, no user can complain that they "pay a lot for almost nothing". From a collective point of view, asking everyone to pay $1/n$ of the cost of meeting the needs of all is justified by solidarity—the idea, not the axiom—and by the fact that consuming one's needs is a legitimate pursuit.

Remark 4. Recalling Expression (20), notice that x^{UF} is also a special variant of the Conditional Equality solutions that consists in choosing zero responsibility as a reference: $\mathbf{q_0} = \bar{\mathbf{q}}$. This implies charging households the same fee to meet their own needs, whatever those needs may be. Should they choose to consume more, they would bear the consequences according to the cost-sharing rule in effect. From a normative standpoint, choosing "zero responsibility" as the reference level is ethically meaningful. In a context where consumers have needs, calibrating the rate function relative to the satisfaction of everyone's needs is normatively appealing.

Theorems 1, 2 and 3 illustrate the tension between compensating for differences in needs and holding users responsible for their consumption.Table 1 summarizes the frontier of comptability between the various versions of these desiderata.

6 Specifying the cost sharing rule, ξ

Having addressed the interplay between compensation and responsibility in Section 5, we now tackle the issue of interdependence through the cost function. It is this very interdependence that prevents convention solutions from guaranteeing a balanced budget (recall, in particular, Propositions 3 and 2).

Formally, up until now, we have remained silent on the shape of the cost sharing rule, ξ, which handles the responsibility portion of the rate function. In principle, any cost-sharing rule could work. However, we will highlight a well-know one that which possesses desirable incentives and fairness properties: the serial cost sharing rule (Moulin and Shenker, 1992).

6.1 The serial cost sharing rule

In the classical setting, in which needs are absent, consumer i simply demands q_i units. Without loss of generality, suppose $q_1 \leq q_2 \leq ... \leq q_n$. The serial cost sharing rule writes:

$$\sigma_i(\mathbf{q}) = \frac{1}{n} C(nq_1) + \sum_{k=2}^{i} \frac{1}{n-k+1} \left[C\left(Q^k\right) - C\left(Q^{k-1}\right) \right], \tag{21}$$

with $Q^k = \sum_{j<k} q_j + (n-k+1) q_k$.[18]

Inuitively speaking, the serial rule splits cost increments equally among those who are responsible for reaching a given demand level. It is very much in the spirit of Littlechild and Owen's cost allocation for a landing strip in airport problems (Littlechild and Owen, 1973): all aircraft carriers use at least the portion of strip required by the smallest planes and share this cost equally. Then, the extra length of strip required to accommodate larger planes is split equally among those larger aircraft carriers, with no additional cost to the smallest carriers. Just like Littlechild and Owen's cost allocation for landing strips is the Shapley value of the airport game, the Moulin and Shenker serial cost sharing rule is also the Shapley value of the appropriate cooperative game (Albizuri et al., 2003). Hence, the serial cost sharing rule inherits a number of fairness properties, which we briefly discuss.

When costs are convex, reflecting negative externalities in consumption, σ protects small consumer from the high marginal cost imposed by large consumers. This is reflected by a property of Independence of Higher Demands: an individual's bill is unaffected by changes in the consumption of larger consumers. The serial rule achieves this property by effectively capping all demands larger than the individual whose cost share is being computed (Moulin and Shenker, 1992).

When costs are concave, reflecting positive externalities, σ is one of the few sharing rules to pass both the No Envy test—no consumer prefers another's consumption-bill combination to their own—and the Stand Alone test—no consumer (or combination of

[18]Naturally, if $i = 1$, the summation term is zero.

consumers) is asked to pay more than the cost of serving them alone—in equilibrium (Moulin, 1996). In addition, the serial rule passes the Unanimity test: $x_i \geq C(nq_i)/n$; my cost share is at least what my fair share would be if every other consumer had the same demand as my own (Moulin, 1996).

There is a clear trade-off between fairness and efficiency: it is a well-established fact that no first-best solution passes most equity tests (Vohra, 1992). Nevertheless, the serial rule exhibits robust strategic properties: While not first-best efficient, the equilibrium of the game where consumers simultaneously choose their demand level obtains by elimination of strictly dominated strategies when costs are convex (Moulin and Shenker, 1992).

6.2 Needs-adjusted serial rate functions

In a framework where needs are present, we adapt the serial rule to be based on the individual's responsibility, $r_i(q_i, \bar{q}_i)$, rather than on their raw consumption, q_i. Just like Independence of Higher Demands is a characteristic feature of the (classical) serial rule, we construct the needs-adjusted serial rule from a new axiom of Independence of Higher Responsbility: an individual's bill is unaffected by changes in the responsibility level of consumers with an already higher responsibilty level.

Axiom. ***(Independence of Higher Responsibility, IHR)*** *For all* $(\mathbf{q}, \bar{\mathbf{q}})$ *and* $(\mathbf{q}', \bar{\mathbf{q}}')$ *such that* $\bar{\mathbf{q}}' = \bar{\mathbf{q}}$ *and* $\mathbf{r}' \geq \mathbf{r}$. *For all* $i \in N$, *define* $L(i) = \{j \in N \text{ s.t. } r_j \leq r_i\}$ *the set of consumers with lower responsibility than* i. *Then,*

$$\begin{aligned} & \left\{r'_j = r_j \text{ for all } j \in L(i)\right\} \\ \Longrightarrow \quad & \left\{\xi_j\left(\mathbf{r}', C - C\left(\bar{Q}'\right)\right) = \xi_j\left(\mathbf{r}, C - C\left(\bar{Q}\right)\right) \text{ for all } j \in L(i)\right\}. \end{aligned}$$

Remark 5. Note that for a given profile $(\mathbf{q}, \bar{\mathbf{q}})$, such that $q_i > q_j$ and $\bar{q}_i > \bar{q}_j$ for some i and j, then one can find two functional forms $\tilde{r}$ and $\hat{r}$ such that

$$\tilde{r}(q_i, \bar{q}_i) \geq \tilde{r}(q_j, \bar{q}_j) \quad \text{and} \quad \hat{r}(q_i, \bar{q}_i) < \hat{r}(q_j, \bar{q}_j).$$

Hence, the identity of consumers with a smaller responsibility depends on how responsibility is measured; i.e., upon the specific functional form for r (recall Figure 1).

Also, notice that when needs are zero for all, the framework boils down to the classical cost-sharing framework, where consumers are unambiguously ranked in order of increasing consumption.

Serial Egalitarian Equivalence (x^{SEE})

When combined with **Solidarity** and **SRRN**, the **IHR** requirement characterizes a unique family of solutions, which amounts to applying the serial cost-sharing rule directly to consumption, along with transfers to compensate for differences in needs.

Proposition 5. *A rate function x satisfies **GS**, **SRRN** and **IHR** if and only if $x = x^{SEE}$ where, for a given reference level of needs $\bar{q}_0 > 0$,*

$$\begin{aligned} x_i^{SEE}(\mathbf{q}, \bar{\mathbf{q}}) &= \frac{C(n\bar{q}_0)}{n} + \sum_{k=1}^{i} \frac{1}{n-k+1}\left[C\left(\tilde{Q}^k\right) - C\left(\tilde{Q}^{k-1}\right)\right] \\ &\quad + \left[u_i(q_i, \bar{q}_i) - u_i(q_i, \bar{q}_0)\right] - \frac{1}{n}\sum_{k=1}^{n}\left[u_k(q_k, \bar{q}_k) - u_k(q_k, \bar{q}_0)\right] \end{aligned}$$

for all $i \in N$, where $\tilde{Q}^k = \sum_{l=1}^{k} q_l + (n-k)\, q_k$ with the set of consumers ordered so as to have $q_1 \leq q_2 \leq ... \leq q_n$.

Proof. In Appendix B.1. □

Remark 6. The rate function x^{SEE} is actually independent of the responsibility function, r. It handles differences in needs—through the utility-compensation terms—and differences in consumption—through the cost-sharing terms—separately.

Remark 7. Note that the compensation terms may affect the well-known incentives properties of the serial cost-sharing rule.

Serial Conditional Equality (x^{SCE})

Recall that $r(\cdot, \bar{q}_i)$ maps an consumer's consumption to her responsibility level, given her needs. Define the inverse of this function, $g_i(\cdot) = (r)^{-1}(\cdot, \bar{q}_i)$, which maps a responsibility level to the corresponding consumption level given the needs of the consumer.

When combined with **Shared Responsibility** and **UWRR**, the **IHR** requirement characterizes a unique family of solutions, which amounts to applying the serial cost-sharing rule directly to consumption, along with transfers to compensate for differences in needs.

Proposition 6. *A rate function x satisfies **Shared Responsibility**, **UWRR** and **IHR** if and only if $x = x^{SCE}$ where, for a given reference responsibility level, $r_0 \geq 0$,*

$$x_i^{SCE}(\mathbf{q}, \bar{\mathbf{q}}) = \frac{1}{n} C\left(\hat{Q}^1\right) + \sum_{k=2}^{i} \frac{1}{n-k+1}\left[C\left(\hat{Q}^i\right) - C\left(\hat{Q}^{i-1}\right)\right] \tag{22}$$

$$+u_i\left(q_i^0, \bar{q}_i\right) - \frac{1}{n}\sum_{j \in N} u_j\left(q_j^0, \bar{q}_j\right) \tag{23}$$

for all $i \in N$, where $\hat{Q}^k = \sum_{i=1}^{k-1} q_i + \sum_{i=k}^{n} g_i(r_k)$ with the set of consumers ordered so as to have $r_1 \leq r_2 \leq ... \leq r_n$.

Proof. In Appendix B.2. □

Remark 8. The rate function x^{SCE} depends on the responsibility function, r, through the $g_i(r_k)$ terms. They are the consumption levels at which consumer i would achieve responsibility level r_k.

Remark 9. The equal split $C\left(\bar{Q}\right)/n$ does not appear in Expression (22) because it simplifies with the first term of the serial cost-sharing rule:

$$\xi_i\left(\mathbf{r}, C - C\left(\bar{Q}\right)\right) = \frac{1}{n}\left[C\left(\hat{Q}^1\right) - C\left(\bar{Q}\right)\right] + \sum_{k=2}^{i}\frac{1}{n-k+1}\left[C\left(\hat{Q}^i\right) - C\left(\hat{Q}^{i-1}\right)\right]. \tag{24}$$

The Serial Utility-Free Rate Function (x^{SUF})

When the consumers share a common utility function and the responsibility function, r, reflects the utility derived by the consumers **Shared Responsibility**, **EWER** and **IHR** together characterize a unique solution, which amounts to applying the serial cost-sharing rule to responsibility levels, without any explicit utility adjustments.

Proposition 7. *If $u_i = u \in \Upsilon$, for all $i \in N$ and if $r = \rho \circ u$, for some increasing function $\rho : \mathbb{R} \to \mathbb{R}_+$, the unique rate function satisfying **Shared Responsibility**,*

EWER *and* ***IHR*** *is the following:*

$$x_i^{SUF}(\mathbf{q}, \bar{\mathbf{q}}) = \frac{1}{n} C\left(\hat{Q}^1\right) + \sum_{k=2}^{i} \frac{1}{n-k+1} \left[C\left(\hat{Q}^k\right) - C\left(\hat{Q}^{k-1}\right)\right] \qquad \textit{for all } i \in N, \tag{25}$$

where, for all $k \in N$*,*

$$\hat{Q}^k = \sum_{i=1}^{k-1} q_i + \sum_{i=k}^{n} g_i(r_k), \tag{26}$$

where the set of consumers is ordered so as to have $r_1 \leq r_2 \leq ... \leq r_n$*.*

Proof. In Appendix B.3. □

Remark 10. At first blush, the expressions of x^{SUF} and x^{SEE} may seem similar, with x^{SEE} having an additional compensation term. However, note that consumers are ordered according to their consumption under x^{SEE} but are ordered according to their responsibility level under x^{SUF}. Also, the Q^k's that enter in the cost-sharing portion stand for different aggregate consumption levels. In particular, x^{SEE} applies the serial cost-sharing rule directly on consumption levels, with needs appearing only in the compensation portion. By contrast, x^{SUF} applies the serial cost-sharing rule to responsibility levels which, by design, take individual needs into account.

7 Accounting for needs in practice

In practice, making explicit interpersonal comparisons of needs and consumption would be very difficult and possibly counterproductive. Nevertheless, we show how one can implement the above schemes with realistic informational assumptions (Section 7.1) and provide some illustrative examples (Section 7.2).[19]

7.1 Pricing using aggregate distributions

We now represent the population by a distribution. Assume a finite number of types in the needs dimension corrsponding to, say, household size, and let $\bar{q}_s$ denote the needs of a household of size $s \in S$. The planner does not know each individual's utility function, but has enough information to infer, u_s, the typical utility function

[19]Computations can be found in Appendix C

of a household of type $s \in S$. Let $n_s(q)$ be the density of type-s households with consumption level q and let $N_s(q)$ be the associated cumulative distribution: $N_s(q) = \int_{z=0}^{q} n_s(z)\,dz$. Define $n(q) = \sum_{s\in S} n_s(q)$ and $N(q) = \sum_{s\in S} N_s(q)$. Lastly, $N_s = \int_{z=0}^{\infty} n_s(z)\,dz$ denotes the total number of consumers of size s and $N = \sum_{s\in S} N_s$ the total population.

Given the responsibility function $r \equiv r(q, \bar{q}_s)$, define $n_s^r(\rho)$ the density of type-s households with responsibility level ρ. Let $N_s^r(\rho)$ be the associated cumulative distribution: $N_s^r(\rho) = \int_{z=0}^{\rho} n_s^r(z)\,dz$ and define $N^r(\rho) = \sum_{s\in S} N_s^r(\rho)$. We now define the following continuous counterparts to the quantities $\tilde{Q}$ and $\hat{Q}$ of the previous section, respectively corresponding to the SEE, SCE and SUF schemes:

$$\text{SEE} \quad : \quad \tilde{Q}(q) = \int_0^{\infty} \inf\{q, z\} n(z)\,dz \tag{27}$$

$$\text{SCE and SUF} \quad : \quad \hat{Q}(\rho) = \sum_{s\in S} \left[\int_0^{+\infty} g_s(\inf\{\rho, z\})\, n_s^r(z)\,dz \right] \tag{28}$$

with $g_s(\cdot) \equiv r^{-1}(\cdot, \bar{q}_s)$.

With this notation, the expression for x^{SEE} for reference needs $\bar{q}_0$ becomes:

$$x^{SEE}(q,s) = \frac{C(n\bar{q}_0)}{n} \tag{29}$$

$$+ \int_{z=0}^{q} C'\left(\tilde{Q}(z)\right) dz \tag{30}$$

$$+ \left[u_s(q, \bar{q}_s) - u_s(q, \bar{q}_0)\right] - \frac{1}{N} \sum_{t\in S} \int_{z=0}^{\infty} \left[u_t(z, \bar{q}_t) - u_t(z, \bar{q}_0)\right] n_t(z)\,dz \tag{31}$$

As before, x^{SEE} splits equally the hypothetical cost if all consumed the reference level of needs (29) before applying the classical serial cost-sharing rule (30) and granting a compensation term (31). The latter amounts to granting the difference in utility relative to having the reference level of needs, $\bar{q}_0$, and subracting the average of that difference across the population.

Similarly, the expression for x^{SCE} with reference responsibility level r_0 is then

$$x^{SCE}(\rho, s) = \frac{C\left(\hat{Q}^0\right)}{N} \tag{32}$$

$$+ \int_{z=0}^{\rho} \frac{1}{N - N^r(z)} C'\left(\hat{Q}(z)\right) \frac{d\hat{Q}(z)}{dz} dz \tag{33}$$

$$+ u_s\left(q_s^0, \bar{q}_s\right) - \sum_{t \in S} \frac{N_t}{N} u_t\left(q_t^0, \bar{q}_t\right) \tag{34}$$

where $q_s^0 = g_s(r_0) = r^{-1}(r_0, \bar{q}_s)$ and $\hat{Q}^0 = \sum_{s \in S} N_s q_s^0$. x^{SCE} splits equally the costs of meeting the needs of all (32) before applying the serial cost-sharing rule *modified to apply to responsibility levels* (33) and granting a compensation term (34). The latter is the difference between the (virtual) utility level if the household had responsibility level r_0 and the average of that virtual utility level in the population.

Finally, x^{SUF}, which is x^{SCE} associated with reference responsibility level $r_0 = 0$ (recall Remark 4) is relatively simple:

$$x^{SUF}(\rho, s) = \frac{C\left(\bar{Q}\right)}{N} + \int_{z=0}^{\rho} \frac{1}{N - N^r(z)} C'\left(\hat{Q}(z)\right) \frac{d\hat{Q}(z)}{d\rho} dz. \tag{35}$$

In particular, x^{SUF} does not explicitly depend on household size, s, except through the computation of $\rho = r(q, \bar{q}_s)$.

7.2 Illustrative Examples

To illustrate, we now consider two specific forms for r. In the *absolute responsibility view*, $r(q, \bar{q}_s) \equiv q - \bar{q}_s$, whereas in the *relative responsibility view*, $r(q, \bar{q}_s) \equiv (q - \bar{q}_s)/\bar{q}_s$. If s indeed denotes household size, the former holds households equally responsible for consumption above needs regardless of their size. By contrast, the latter view holds larger households less responsible than smaller households for an identical consumption level above needs. In other words, needs also impact the way consumption beyond them is considered.

Decreasing Returns to Scale : Quadratic Costs

Assume that costs are given by the following quadratic function: $C(Q) = cQ^2/2$. Under absolute responsibility, the serial conditional equality rule with zero responsibility as a reference yields:

$$x^{SUF-abs}(q,s) = \frac{1}{N}\frac{cQ^2}{2} + cQ\left(q - \bar{q}_s - \frac{Q-\bar{Q}}{N}\right). \tag{36}$$

In words, consumers share the total cost equally and are rewarded or penalized for deviation from the average responsibility level. These deviations are valued at marginal cost.

Under relative responsibility, however, marginal consumption is not priced equally across household types. When the empirical distribution of responsibility is identical across types, we obtain the following expression:

$$x^{SUF-rel}(q,s) = \frac{1}{N}\frac{cQ^2}{2} + cQ\frac{\bar{Q}}{N}\left(\frac{q-\bar{q}_s}{\bar{q}_s} - \frac{Q-\bar{Q}}{\bar{Q}}\right). \tag{37}$$

Again, the utility-free rate function charges everyone the average cost and prices deviations from the average responsibility, but this time at the marginal cost of responsibility if needs were equal to $\bar{Q}/N$. Observe that if $\bar{q}_s > \bar{Q}/N$ consumption is priced at less than the marginal cost while the consumption of households with lower-than-average needs ($\bar{q}_s < \bar{Q}/N$) is priced above marginal cost.

While the serial egalitarian equivalent solution, x^{SEE}, and serial conditional equality solution, x^{SCE}, can be computed in a similar fashion, they do not simplify in any meaningful way. This is due to the utility terms, which only simplify in the case of overly simplistic functional forms for utility functions.

Increasing Returns to Scale: Affine Costs

Assume costs are of the form $C(Q) = F + cQ$, with $F, c \in \mathbb{R}_+$. When responsibility is measured by absolute responsibility, the decreasing serial conditional equality rule yields:

$$x^{SUF-abs}(q,s) = \frac{F + c\bar{Q}}{N} + c\left(q - \bar{q}_s\right). \tag{38}$$

In addition to splitting the fixed cost equally, $x^{SUF-abs}$ also splits the cost of the population's needs equally before charging consumers at marginal cost with a rebate equal to the cost of meeting their needs.

Under the relative responsibility view, and if responsibility is identically distributed across types, we obtain:

$$x^{SUF-rel}(q,s) \;=\; \frac{F}{N} + c\frac{1}{\bar{q}_s/\left(\bar{Q}/N\right)}q. \tag{39}$$

As with relative responsibility, the utility-free rate function splits the fixed cost equally. No rebate is granted, however, but consumption is priced at a rate that is inversely proportional to one's needs.

As previously, x^{SEE} and x^{SCE} they do not simplify in any meaningful way.

8 Concluding remarks

By making explicit the distribution of needs in the population, we were able to design sharing rules to account for the essential nature of some public services, like water and electricity. Being able to manage essential services in a way that accounts for the needs of the population is important in the best of times, but is even more crucial when resources become scarce due to climate or geopolitical disruption. As we have shown, simply aiming for affordability, under the assumption that consumers take their own needs into account when making rational purchases, cannot solve the issue. This is because the needs issue goes beyond ensuring that one has enough to meet their own needs: it is an issue of burden sharing that requires a somewhat delicate level of coordination.

The families of sharing rules we exhibit leave freedom to the planner. They may choose a pricing scheme that places more emphasis on compensation, or on reponsibility. Or they may attempt to achieve both by making the extra effort to know their consumers (and then selecting the responsibility function that matches the consumers' utility functions), thereby simplifying the rate function in the process (because the utility-free solution does not require computing utility levels). In addition, the formulae we obtain resemble the familiar two-part tariffs that utilities and municipalities use, yet ours are ethically grounded on adressing needs and guarantee that total costs are covered.

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A Appendix: Section 5 Proofs

A.1 Proof of Proposition 3

Consider a service charge that covers the total cost—$\sum_i t_i(\mathbf{q}) = C(Q)$ for all $\mathbf{q} \in \mathbb{R}^n_+$—and a subsidy that covers the collective cost of meeting the population's needs: $\sum_i s_i(\bar{\mathbf{q}}) = C(\bar{Q})$ for all $\bar{\mathbf{q}} \in \mathbb{R}^n_+$.

Consider a profile $(\mathbf{q}, \bar{\mathbf{q}}) \in \mathbb{R}^{2n}_+$, and an alternative needs profile where only consumer 1's needs have changed: $\bar{\mathbf{q}}' = (\bar{q}_1', \bar{q}_2, ..., \bar{q}_n) \in \mathbb{R}^n_+$ with $\bar{q}_1' \neq \bar{q}_1$. The welfare consequence to consumer 1's of her change in needs is equal to:

$$[u_1(q_1, \bar{q}_1') - (t_1(\mathbf{q}) - s_1(\bar{\mathbf{q}}'))] - [u_1(q_1, \bar{q}_1) - (t_1(\mathbf{q}) - s_1(\bar{\mathbf{q}}))] = [u_1(q_1, \bar{q}_1') + s_1(\bar{\mathbf{q}}')] - [u_1(q_1, \bar{q}_1) + s_1 \tag{40}$$

$$= [u_1(q_1, \bar{q}_1') - u_1(q_1, \bar{q}_1)] + [s_1(\bar{\mathbf{q}}') - s_1 \tag{41}$$

Because the purpose of the monetary subsidy is to avoid consumers being penalized (resp. privileged) for having higher (resp. lower) needs, the latter welfare change should equal zero, so that:

$$s_1(\bar{\mathbf{q}}') - s_1(\bar{\mathbf{q}}) = u_1(q_1, \bar{q}_1) - u_1(q_1, \bar{q}_1'). \tag{42}$$

The latter must hold for any $\bar{\mathbf{q}}, \bar{\mathbf{q}}'$and any $q_1 \geq 0$. In particular, consider $q_1' \neq q_1$. It follows that:

$$u_1(q_1', \bar{q}_1) - u_1(q_1', \bar{q}_1') = u_1(q_1, \bar{q}_1) - u_1(q_1, \bar{q}_1') \tag{43}$$

for all $q_1, q_1', \bar{q}_1 \bar{q}_1' \geq 0$. Hence, defining $\delta = q_1' - q_1$, and dividing through by δ, we obtain:

$$\frac{u_1(q_1 + \delta, \bar{q}_1) - u_1(q_1, \bar{q}_1)}{\delta} = \frac{u_1(q_1 + \delta, \bar{q}_1') - u_1(q_1, \bar{q}_1')}{\delta}. \tag{44}$$

Taking the limit as $\delta \to 0$ yields that $\partial u_1 / \partial q_1$ is independent from its second argument. It follows that u_1 is additively separable: there exist two non-decreasing functions, v_1 and w_1, such that

$$u_1(q_1, \bar{q}_1) \equiv v_1(q_1) - w_1(\bar{q}_1). \tag{45}$$

Recall that we normalized utility functions so that $u_i(\bar{q}_i, \bar{q}_i) = 0$ for all $\bar{q}_i$. It follows that the functions v_1 and w_1 must be identical:

$$u_1(q_1, \bar{q}_1) \equiv v_1(q_1) - v_1(\bar{q}_1). \tag{46}$$

Plugging back into (42), we get:

$$s_1(\bar{\mathbf{q}}') - s_1(\bar{\mathbf{q}}) = v_1(\bar{q}_1') - v_1(\bar{q}_1). \tag{47}$$

Upon noticing that the right-hand side does not depend on $\bar{q}_2, \bar{q}_3, ..., \bar{q}_n$, we can write s_1 as the sum of a function of $\bar{q}_1$ and of a function of $(\bar{q}_2, \bar{q}_3, ..., \bar{q}_n)$: $s_1(\bar{\mathbf{q}}) = f_1(\bar{q}_1) + g_1(\bar{q}_2, \bar{q}_3, ..., \bar{q}_n)$. Plugging back into (47),

$$f_1(\bar{q}_1') - f_1(\bar{q}_1) = v_1(\bar{q}_1') - v_1(\bar{q}_1), \tag{48}$$

which rearranges into

$$f_1(\bar{q}_1') - v_1(\bar{q}_1') = f_1(\bar{q}_1) - v_1(\bar{q}_1). \tag{49}$$

Recall that the latter equality must hold for all values of $\bar{q}_1$ and $\bar{q}_1'$. This establishes that there exists some $k_1 \in \mathbb{R}$ such that $f_1(\bar{q}_1) \equiv v_1(\bar{q}_1) + k_1$ for all $\bar{q}_1$. Hence, the subsidy s_1 must be of the form:

$$s_1(\bar{\mathbf{q}}) = v_1(\bar{q}_1) + g_1(\bar{q}_2, \bar{q}_3, ..., \bar{q}_n). \tag{50}$$

It follows that consumer i's net utility level is equal to:

$$u_1(q_1, \bar{q}_1) - (t_1(\mathbf{q}) - s_1(\bar{\mathbf{q}})) = v_1(q_1) - t_1(\mathbf{q}) + g_1(\bar{q}_2, \bar{q}_3, ..., \bar{q}_n), \tag{51}$$

which is actually entirely independent of consumer 1's own needs, $\bar{q}_1$.

A.2 Proof of Proposition 4

From (50) in the proof of the previous proposition and recalling (18), we obtain:

$$\sum_i v_i(\bar{q}_i) + \sum_i g_i(\bar{\mathbf{q}}_{-i}) = C(\bar{Q}) \tag{52}$$

for all $\bar{\mathbf{q}}$. Assume an increase in consumer 1's needs while those of others are unchanged, so that we shift from $\bar{\mathbf{q}} = (\bar{q}_1, \bar{q}_2, ..., \bar{q}_n)$ to $\bar{\mathbf{q}}' = (\bar{q}_1', \bar{q}_2, ..., \bar{q}_n)$ with $\bar{q}_1' \neq \bar{q}_1$ and consider the impact of the shift from $\bar{\mathbf{q}}$ to $\bar{\mathbf{q}}'$ on the welfare of all consumers. It is apparent from Expression (51) that consumer 1's net utility is unaffected by this change. Hence, if the aim is share the burden equally across all consumers, including Consumer 1, then it must be that all consumers see their net utility unchanged. We turn to this case later in the proof.

Consider first the milder requirement that the net utility of consumers *other than 1* should be affected equally. From (51), because the consumption vector is unchanged, this implies that the change in the amount of subsidy should be the same: for all $i, j \neq 1$,

$$g_i \left(\bar{\mathbf{q}}_{-i}'\right) - g_i \left(\bar{\mathbf{q}}_{-i}\right) = g_j \left(\bar{\mathbf{q}}_{-j}'\right) - g_j \left(\bar{\mathbf{q}}_{-j}\right). \tag{53}$$

Subtracting Expression (52) to budget balance over the profile $\bar{\mathbf{q}}$ from that at profile $\bar{\mathbf{q}}'$, and using (53) we obtain:

$$v_1 \left(\bar{q}_1'\right) - v_1 \left(\bar{q}_1\right) + (n-1) \left[g_2 \left(\bar{\mathbf{q}}_{-2}'\right) - g_2 \left(\bar{\mathbf{q}}_{-2}\right)\right] = C\left(\bar{Q}'\right) - C\left(\bar{Q}\right). \tag{54}$$

This rewrites into:

$$g_2 \left(\bar{\mathbf{q}}_{-2}'\right) - g_2 \left(\bar{\mathbf{q}}_{-2}\right) = \frac{1}{n-1} \left[C\left(\bar{Q}'\right) - v_1 \left(\bar{q}_1'\right) - \left(C\left(\bar{Q}\right) - v_1 \left(\bar{q}_1\right)\right)\right]. \tag{55}$$

Notice that the left-hand side is independent of $\bar{q}_2$, implying that it must also be the case for the right-hand side.

Hence, consider any $\bar{q}_2' \neq \bar{q}_2$. From (55), the following equality must hold:

$$C\left(\bar{Q}' + \bar{q}_2' - \bar{q}_2\right) - v_1 \left(\bar{q}_1'\right) - \left(C\left(\bar{Q} + \bar{q}_2' - \bar{q}_2\right) - v_1 \left(\bar{q}_1\right)\right) = C\left(\bar{Q}'\right) - v_1 \left(\bar{q}_1'\right) - \left(C\left(\bar{Q}\right) - v_1 \left(\bar{q}_1\right)\right) \tag{56}$$

and, therefore,

$$C\left(\bar{Q}' + \bar{q}_2' - \bar{q}_2\right) - C\left(\bar{Q} + \bar{q}_2' - \bar{q}_2\right) = C\left(\bar{Q}'\right) - C\left(\bar{Q}\right). \tag{57}$$

Denoting $\delta_1 = \bar{q}_1' - \bar{q}_1$ and $\delta_2 = \bar{q}_2' - \bar{q}_2$, the latter expression becomes:

$$C\left(\bar{Q} + \delta_1 + \delta_2\right) - C\left(\bar{Q} + \delta_2\right) = C\left(\bar{Q} + \delta_1\right) - C\left(\bar{Q}\right) \tag{58}$$

and must hold for any δ_1, δ_2. Dividing by δ_1 and taking the limit for $\delta_1 \to 0$ yields

$$C'\left(\bar{Q} + \delta_2\right) = C'\left(\bar{Q}\right). \tag{59}$$

Because the latter must hold for all δ_2, it follows that C' is a constant function, meaning that C is affine. Hence, there exist $c \geq 0$ and k such that

$$C\left(Q\right) \equiv cQ + k. \tag{60}$$

Remark 11. Notice that the requirement that C be affine is the result of the milder requirement that consumers other than i share the burden equally among them. It is not driven by the requirement that consumers other than i are unaffected, which we now turn to.

Plugging back into Expression (55), we get:

$$g_2\left(\bar{\mathbf{q}}'_{-2}\right) - g_2\left(\bar{\mathbf{q}}_{-2}\right) = \frac{1}{n-1}\left[c\bar{Q}' + k - v_1\left(\bar{q}'_1\right) - \left(c\bar{Q} + k - v_1\left(\bar{q}_1\right)\right)\right] \tag{61}$$

$$= \frac{1}{n-1}\left[c\delta_1 - \left(v_1\left(\bar{q}'_1\right) - v_1\left(\bar{q}_1\right)\right)\right]. \tag{62}$$

Dividing through by δ_1 and taking the limit for $\delta_1 \to 0$ yields:

$$\frac{\partial g_2}{\partial \bar{q}_1} = \lim_{\delta_1 \to 0} \frac{g_2\left(\bar{\mathbf{q}}'_{-2}\right) - g_2\left(\bar{\mathbf{q}}_{-2}\right)}{\delta_1} \tag{63}$$

$$= \frac{1}{n-1}\left[c - v'_1\left(\bar{q}_1\right)\right]. \tag{64}$$

Therefore, v_1 and C are identical up to a constant; hence, $v_1\left(q_1\right) \equiv cq_1 + k_1$. Clearly, the above reasoning applies to all consumers $i \in N$ so that

$$u_i\left(q_i, \bar{q}_i\right) \equiv v_i\left(q_i\right) - v_i\left(\bar{q}_i\right) \equiv c\left(q_i - \bar{q}_i\right). \tag{65}$$

A.3 Proof of Theorem 1

Let $\bar{q}_0 \in \mathbb{R}_+$ be a reference level of needs and denote by $\bar{\mathbf{q}}_0 = \left(\bar{q}_0, \bar{q}_0, ..., \bar{q}_0\right) \in \mathbb{R}^n_+$ the associated reference vector. Let $\left(\mathbf{q}, \bar{\mathbf{q}}\right) \in \mathbb{R}^{2n}_+$. By budget balance and anonymity,

$$x_i\left(\bar{\mathbf{q}}_0, \bar{\mathbf{q}}_0\right) = \frac{C(n\bar{q}_0)}{n}. \tag{66}$$

By **SRRN**,

$$x_i(\mathbf{q}, \bar{\mathbf{q}}_0) - x_i(\bar{\mathbf{q}}_0, \bar{\mathbf{q}}_0) = \xi_i(\mathbf{r}_0, C - C(n\bar{q}_0)) \qquad \text{for all } i \in N, \tag{67}$$

where $r_{0,i} = r(q_i, \bar{q}_0)$ for all i.

Define $\bar{\mathbf{q}}_0^1 = (\bar{q}_1, \bar{q}_0, ..., \bar{q}_0)$. Applying **Solidarity** between $(\mathbf{q}, \bar{\mathbf{q}}_0)$ and $(\mathbf{q}, \bar{\mathbf{q}}_0^1)$ yields, for all $j \neq 1$:

$$u_1(q_1, \bar{q}_1) - x_1^1 - \left[u_1(q_1, \bar{q}_0) - x_1^0\right] \;=\; u_j(q_j, \bar{q}_0) - x_j^1 - \left[u_j(q_j, \bar{q}_0) - x_j^0\right] \tag{68}$$

where $x_j^0 = x_j(\mathbf{q}, \bar{\mathbf{q}}_0)$ and $x_j^1 = x_j(\mathbf{q}, \bar{\mathbf{q}}_0^1)$ for all $j \in N$. This yields:

$$x_1^1 - x_1^0 = u_1(q_1, \bar{q}_1) - u_1(q_1, \bar{q}_0) + x_j^1 - x_j^0, \tag{69}$$

for all $j \neq 1$. Summing up over all $j \neq 1$ yields:

$$(n-1)\left(x_1^1 - x_1^0\right) = (n-1)\left[u_1(q_1, \bar{q}_1) - u_1(q_1, \bar{q}_0)\right] + \sum_{j \neq 1}\left(x_j^1 - x_j^0\right). \tag{70}$$

Adding $x_1^1 - x_1^0$ to both sides and noticing that $\sum\left(x_j^1 - x_j^0\right) = 0$ by budget balance yields:

$$\begin{cases} x_1^1 - x_1^0 & = \frac{n-1}{n}\left[u_1(q_1, \bar{q}_1) - u_1(q_1, \bar{q}_0)\right] \\ x_j^1 - x_j^0 & = -\frac{1}{n}\left[u_1(q_1, \bar{q}_1) - u_1(q_1, \bar{q}_0)\right] \qquad \forall j \neq 1. \end{cases} \tag{71}$$

Similarly, applying **Solidarity** to profiles $\left(\mathbf{q}, \bar{\mathbf{q}}_0^k\right)$ where $\bar{\mathbf{q}}_0^k = (\bar{q}_1, \bar{q}_2, ..., \bar{q}_k, \bar{q}_0, ..., \bar{q}_0)$, successively leads to the following expression, for all iterations, $k = 1, ..., n$, and all consumers $1 \leq i \leq k \leq j \leq n$:

$$u_i(q_i, \bar{q}_i) - x_i^k - u_i(q_i, \bar{q}_i) + x_i^{k-1} \tag{72}$$

$$= \; u_k(q_k, \bar{q}_k) - x_k^k - u_k(q_k, \bar{q}_0) + x_k^{k-1} \tag{73}$$

$$= \; u_j(q_j, \bar{q}_0) - x_j^k - u_j(q_j, \bar{q}_0) + x_j^{k-1} \tag{74}$$

Hence, for all $k = 1, ..., n$, and all consumers $1 \leq i \leq k \leq j \leq n$:

$$\begin{aligned} & x_i^{k-1} - x_i^k & (75) \\ = \; & u_k\left(q_k, \bar{q}_k\right) - u_k\left(q_k, \bar{q}_0\right) + x_k^{k-1} - x_k^k & (76) \\ = \; & x_j^{k-1} - x_j^k & (77) \end{aligned}$$

By budget balance, $\sum_j \left(x_j^k - x_j^{k-1}\right) = 0$, yielding:

$$x_k^k - x_k^{k-1} = \frac{n-1}{n}\left[u_k\left(q_k, \bar{q}_k\right) - u_k\left(q_k, \bar{q}_0\right)\right] \quad (78)$$

$$x_j^k - x_j^{k-1} = -\frac{1}{n}\left[u_k\left(q_k, \bar{q}_k\right) - u_k\left(q_k, \bar{q}_0\right)\right] \qquad \text{for all } j \neq k. \quad (79)$$

Summing up over all iterations k yields the following:

$$\begin{aligned} x_1^n - x_1^0 &= \sum_{k>1}^{n}\left(x_1^k - x_1^{k-1}\right) + x_1^1 - x_1^0 & (80) \\ &= -\frac{1}{n}\sum_{j>1}^{n}\left[u_j\left(q_j, q_j\right) - u_j\left(q_j, \bar{q}_0\right)\right] + \left(\frac{n-1}{n}\right)\left[u_1\left(q_1, \bar{q}_1\right) - u_1\left(q_1, \bar{q}_0\right)\right] & (81) \\ &= \left[u_1\left(q_1, \bar{q}_1\right) - u_1\left(q_1, \bar{q}_0\right)\right] - \frac{1}{n}\sum_{j=1}^{n}\left[u_j\left(q_j, \bar{q}_j\right) - u_j\left(q_j, \bar{q}_0\right)\right]. & (82) \end{aligned}$$

Likewise, for all $i \in N$:

$$x_i^n - x_i^0 = \left[u_i\left(q_i, \bar{q}_i\right) - u_i\left(q_i, \bar{q}_0\right)\right] - \frac{1}{n}\sum_{j=1}^{n}\left[u_j\left(q_j, \bar{q}_j\right) - u_j\left(q_j, \bar{q}_0\right)\right]. \quad (83)$$

Finally, upon noticing that $x_i^n = x\left(\mathbf{q}, \bar{\mathbf{q}}\right)$ and $x_i^0 = x_i\left(\mathbf{q}, \bar{\mathbf{q}}_0\right)$, Expression (67) yields:

$$\begin{aligned} x_i\left(\mathbf{q}, \bar{\mathbf{q}}\right) &= \xi_i\left(\mathbf{r}_0, C - C\left(n\bar{q}_0\right)\right) + x_i\left(\bar{\mathbf{q}}_0, \bar{\mathbf{q}}_0\right) & (84) \\ &\quad + \left[u_i\left(q_i, \bar{q}_i\right) - u_i\left(q_i, \bar{q}_0\right)\right] - \frac{1}{n}\sum_{j=1}^{n}\left[u_j\left(q_j, \bar{q}_j\right) - u_j\left(q_j, \bar{q}_0\right)\right]. & (85) \end{aligned}$$

Expression (66) yields the result.

A.4 Proof of tightness of the characterization of EE by Solidarity and SRRN

Let $\bar{q}_1 > \bar{q}_0$, we show that no solution can satisy **Solidarity** along with **SRRN** simultaneously for two distinct reference levels of needs, $\bar{q}_0$ and $\bar{q}_1$. Let x^{EE} be the egalitarian equivalent solution defined relative to reference needs level $\bar{q}_0 \geq 0$ and consider a profile $(\mathbf{q}, \bar{\mathbf{q}}_\mathbf{1}) \in \mathbb{R}_+^{2n}$ such that $\bar{\mathbf{q}}_\mathbf{1} = (\bar{q}_1, \bar{q}_1, ..., \bar{q}_1)$. Then:

$$\begin{aligned} x_i^{EE}(\mathbf{q}, \bar{\mathbf{q}}_\mathbf{1}) - x_i^{EE}(\bar{\mathbf{q}}_\mathbf{1}, \bar{\mathbf{q}}_\mathbf{1}) \quad = \quad & \frac{C(n\bar{q}_0)}{n} + \xi_i(\mathbf{r}_0, C - C(n\bar{q}_0)) \\ & + [u_i(q_i, \bar{q}_1) - u_i(q_i, \bar{q}_0)] - \frac{1}{n}\sum_{k=1}^{n}[u_k(q_k, \bar{q}_1) - u_k(q_k, \bar{q}_0)] \\ & - \left(\frac{C(n\bar{q}_0)}{n} + \xi_i(\bar{\mathbf{r}}_0, C - C(n\bar{q}_0)) \ldots \right. \\ & \left. \ldots + [u_i(\bar{q}_1, \bar{q}_1) - u_i(\bar{q}_1, \bar{q}_0)] - \frac{1}{n}\sum_{k=1}^{n}[u_k(\bar{q}_1, \bar{q}_1) - u_k(\bar{q}_1, \bar{q}_0)] \right) \end{aligned} \tag{86}$$

where $\bar{\mathbf{r}}_0 \equiv (r(\bar{q}_1, \bar{q}_0), r(\bar{q}_1, \bar{q}_0), ..., r(\bar{q}_1, \bar{q}_0)) \in \mathbb{R}_+^n$. Hence, upon noticing that $\xi_i(\bar{\mathbf{r}}_0, C - C(n\bar{q}_0)) = \frac{1}{n}(C(n\bar{q}_1) - C(n\bar{q}_0))$ by anonymity, Expression (86) simplifies into:

$$\begin{aligned} x_i^{EE}(\mathbf{q}, \bar{\mathbf{q}}_1) - x_i^{EE}(\bar{\mathbf{q}}_1, \bar{\mathbf{q}}_1) \quad = \quad & \xi_i(\mathbf{r}_0, C - C(n\bar{q}_0)) - \frac{1}{n}(C(n\bar{q}_1) - C(n\bar{q}_0)) \\ & + \{u_i(q_i, \bar{q}_1) - u_i(\bar{q}_1, \bar{q}_1) - [u_i(q_i, \bar{q}_0) - u_i(\bar{q}_1, \bar{q}_0)]\} \\ & - \frac{1}{n}\sum_{k=1}^{n}\{u_k(q_k, \bar{q}_1) - u_k(q_k, \bar{q}_0) - [u_k(\bar{q}_1, \bar{q}_1) - u_k(\bar{q}_1, \bar{q}_0)]\} \end{aligned} \tag{87}$$

The above expression reveals that $x_i^{EE}(\mathbf{q}, \bar{\mathbf{q}}_1) - x_i^{EE}(\bar{\mathbf{q}}_1, \bar{\mathbf{q}}_1)$ depends on u_i, hence cannot be driven only by the cost sharing function ξ. In other words, it cannot be the case that:

$$x_i^{EE}(\mathbf{q}, \bar{\mathbf{q}}_1) - x_i^{EE}(\bar{\mathbf{q}}_1, \bar{\mathbf{q}}_1) = \xi_i(\mathbf{r}_1, C - C(n\bar{q}_1)),$$

as required by **SRRN** using $\bar{q}_1$ as reference level of needs.

A.5 Proof of Theorem 2

Let $r_0 \in \mathbb{R}_+$ be a reference responsibility level and let $(\mathbf{q}^0, \bar{\mathbf{q}}) \in \mathbb{R}_+^{2n}$ be such that,

$$r\left(q_i^0, \bar{q}_i\right) = r_0, \qquad \text{for all } i \in N. \tag{88}$$

By **UWRR**,

$$u_i\left(q_i^0, \bar{q}_i\right) - x_i\left(\mathbf{q}^0, \bar{\mathbf{q}}\right) = u_j\left(q_j^0, \bar{q}_j\right) - x_j\left(\mathbf{q}^{\mathbf{0}}, \bar{\mathbf{q}}\right), \qquad \text{for all } i, j \in N. \tag{89}$$

Hence, for all $i \in N$,

$$x_i\left(\mathbf{q}^0, \bar{\mathbf{q}}\right) = u_i\left(q_i^0, \bar{q}_i\right) - \frac{1}{n}\sum_{j \in N}\left[u_j\left(q_j^0, \bar{q}_j\right) - x_j\left(\mathbf{q}^{\mathbf{0}}, \bar{\mathbf{q}}\right)\right], \tag{90}$$

$$= \frac{C\left(Q^0\right)}{n} + u_i\left(q_i^0, \bar{q}_i\right) - \frac{1}{n}\sum_{j \in N} u_j\left(q_j^0, \bar{q}_j\right), \tag{91}$$

where $Q^0 \equiv \sum_{j \in N} q_j^0$ and the first term obtains by budget balance—as $\sum_j x_j\left(\mathbf{q}^0, \bar{\mathbf{q}}\right) = C\left(Q^0\right)$.

Applying **Shared Responsibility** between profiles $(\mathbf{q}^0, \bar{\mathbf{q}})$ and $(\bar{\mathbf{q}}, \bar{\mathbf{q}})$ yields:

$$x_i\left(\mathbf{q}^0, \bar{\mathbf{q}}\right) - x_i\left(\bar{\mathbf{q}}, \bar{\mathbf{q}}\right) \;=\; \xi_i\left(\mathbf{r}_0, C - C(\bar{Q})\right). \tag{92}$$

Hence, by symmetry of ξ,

$$x_i\left(\bar{\mathbf{q}}, \bar{\mathbf{q}}\right) = x_i\left(\mathbf{q}^0, \bar{\mathbf{q}}\right) - \frac{C\left(Q^0\right) - C\left(\bar{Q}\right)}{n}. \tag{93}$$

Applying now **Shared Responsibility** between profiles $(\bar{\mathbf{q}}, \bar{\mathbf{q}})$ and $(\mathbf{q}, \bar{\mathbf{q}})$ yields:

$$x_i\left(\mathbf{q}, \bar{\mathbf{q}}\right) - x_i\left(\bar{\mathbf{q}}, \bar{\mathbf{q}}\right) \;=\; \xi_i\left(\mathbf{r}, C - C(\bar{Q})\right). \tag{94}$$

Thus,

$$x_i(\mathbf{q},\bar{\mathbf{q}}) = \xi_i\left(\mathbf{r}, C - C(\bar{Q})\right) + x_i(\bar{\mathbf{q}},\bar{\mathbf{q}}) \tag{95}$$

$$= \xi_i\left(\mathbf{r}, C - C(\bar{Q})\right) + x_i\left(\mathbf{q}^0,\bar{\mathbf{q}}\right) - \frac{C\left(Q^0\right) - C\left(\bar{Q}\right)}{n} \tag{96}$$

$$= \xi_i\left(\mathbf{r}, C - C(\bar{Q})\right) + \frac{C\left(\bar{Q}\right)}{n} + u_i\left(q_i^0,\bar{q}_i\right) - \frac{1}{n}\sum_{j\in N} u_j\left(q_j^0,\bar{q}_j\right). \tag{97}$$

A.6 Proof of Theorem 3

In order to prove Theorem 3, we first establish the conditions under which x^{CE} satisfies a stronger version of compensation than **UWRR**, which consists in equalizing well-beings whenever all share the same responsiblity level (though not necessarily the reference responsibility level). Formally:

Axiom. ***(Uniform Well-being for Uniform Responsibility, UWUR)***

$$[r_i = r_j, \forall i,j \in N] \implies [u_i(q_i,\bar{q}_i) - x_i = u_j(q_j,\bar{q}_j) - x_j, \forall i,j \in N]$$

Lemma 1. x^{CE} *does not satisfy* ***UWUR*** *unless the following two assertions are true:*
(1) all consumers share a common utility function; i.e., $u_i = u \in \Upsilon$*, for all* $i \in N$*,*
(2) the responsibility function co-varies with consumers utility; i.e., $r = \rho \circ u$*, for some increasing function* $\rho : \mathbb{R} \to \mathbb{R}_+$*.*

Proof. Let $(\mathbf{q}^0,\bar{\mathbf{q}}) \in \mathbb{R}^{2n}_+$ and $(\mathbf{q}^1,\bar{\mathbf{q}}) \in \mathbb{R}^{2n}_+$ be two profiles associated respectively with the uniform responsibility profiles $\mathbf{r^0} = (r_0, r_0, ..., r_0)$ and $\mathbf{r^1} = (r_1, r_1, ..., r_0)$ with $r_1 \neq r_0$. Suppose that x satisfies **UWUR**, so that it satisfies in particular **UWRR** for the reference responsibility level r_0. If it does also satisfy **Shared Responsibility,** Theorem 2 implies that it can be written as

$$x_i(\mathbf{q},\bar{\mathbf{q}}) = \frac{C\left(\bar{Q}\right)}{n} + \xi_i\left(\mathbf{r}, C - C(\bar{Q})\right) + u_i\left(q_i^0,\bar{q}_i\right) - \frac{1}{n}\sum_{j\in N} u_j\left(q_j^0,\bar{q}_j\right), \qquad \text{for all } i \in N. \tag{98}$$

This says in particular that, when $\mathbf{q} = \mathbf{q^1}$, we have:

$$x_i\left(\mathbf{q^1}, \bar{\mathbf{q}}\right) = \frac{C\left(\bar{Q}\right)}{n} + \xi_i\left(\mathbf{r}^1, C - C(\bar{Q})\right) + u_i\left(q_i^0, \bar{q}_i\right) - \frac{1}{n}\sum_{j\in N} u_j\left(q_j^0, \bar{q}_j\right), \qquad \text{for all } i \in N. \tag{99}$$

By symmetry of ξ, we have $\xi_i\left(\mathbf{r}^1, C - C\left(\bar{Q}\right)\right) = \left[C\left(Q^1\right) - C\left(\bar{Q}\right)\right]/n$, for all $i \in N$, with $Q^1 = \sum_j q_j^1$, so that

$$x_i\left(\mathbf{q^1}, \bar{\mathbf{q}}\right) = \frac{C\left(Q^1\right)}{n} + u_i\left(q_i^0, \bar{q}_i\right) - \frac{1}{n}\sum_{j\in N} u_j\left(q_j^0, \bar{q}_j\right), \qquad \text{for all } i \in N. \tag{100}$$

Because x also satisfies **UWRR** for the reference responsibility level r_1 (to which $\mathbf{q^1}$ is associated), it must be the case that

$$u_i\left(q_i^1, \bar{q}_i\right) - x_i\left(\mathbf{q^1}, \bar{\mathbf{q}}\right) = u_j\left(q_j^1, \bar{q}_j\right) - x_j\left(\mathbf{q^1}, \bar{\mathbf{q}}\right), \qquad \text{for all } i, j \in N. \tag{101}$$

From the expression of $x_i\left(\mathbf{q^1}, \bar{\mathbf{q}}\right)$ established above, we must have

$$u_i\left(q_i^1, \bar{q}_i\right) - u_i\left(q_i^0, \bar{q}_i\right) = u_j\left(q_j^1, \bar{q}_j\right) - u_j\left(q_j^0, \bar{q}_j\right), \qquad \text{for all } i, j \in N. \tag{102}$$

This implies in turn that

$$u_i\left(q_i^1, \bar{q}_i\right) - u_i\left(q_i^0, \bar{q}_i\right) = \frac{1}{n}\sum_{j\in N}\left[u_j\left(q_j^1, \bar{q}_j\right) - u_j\left(q_j^0, \bar{q}_j\right)\right], \qquad \text{for all } i \in N. \tag{103}$$

This must be true for any responsibility level r^0 and r^1 and the associated profiles $(\mathbf{q}^0, \bar{\mathbf{q}}) \in \mathbb{R}_+^{2n}$ and $(\mathbf{q}^1, \bar{\mathbf{q}}) \in \mathbb{R}_+^{2n}$. Thus, by setting $r_1 = 0$, so that $u_i\left(q_i^1, \bar{q}_i\right) = 0$ for all i—and thus considering the associated profile $(\bar{\mathbf{q}}, \bar{\mathbf{q}}) \in \mathbb{R}_+^{2n}$—we obtain that, for **Shared Responsibility** and **UWUR** to be compatible, the utility functions must be such that

$$u_i\left(q_i^0, \bar{q}_i\right) = \frac{1}{n}\sum_{j\in N} u_j\left(q_j^0, \bar{q}_j\right) \tag{104}$$

for all $i \in N$ and for all profiles $(\mathbf{q}^0, \bar{\mathbf{q}}) \in \mathbb{R}_+^{2n}$ such that

$$r\left(q_i^0, \bar{q}_i\right) = r_0, \qquad \text{for all } i \in N. \tag{105}$$

Now fix r^0 and $\bar{\mathbf{q}}$ and define, for all $i \in N$, $q\left(r_0, \bar{q}_i\right) = \{q \in \mathbb{R}_+ | r\left(q, \bar{q}_i\right) = r_0\}$. By continuity and strict monotonicity of r, $q(r_0, \bar{q}_i)$ is a singleton and $(r_0, \bar{q}_i) \mapsto q(r_0, \bar{q}_i)$ defines a continuous function that is increasing in its first argument. Also, define $u_0 = \frac{1}{n}\sum_{j \in N} u_j(q(r_0, \bar{q}_j), \bar{q}_j)$.

Expression (104) can be rewritten as

$$\frac{n-1}{n} u_i(q(r_0, \bar{q}_i), \bar{q}_i) = \frac{1}{n} \sum_{j \in N \setminus i} u_j(q(r_0, \bar{q}_j), \bar{q}_j). \tag{106}$$

Because the right-hand side is independent of $\bar{q}_i$, it must be that $u_i(q_i^0, \bar{q}_i)$ is also independent of $\bar{q}_i$, implying that utility is entirely determined by the responsibility level, r_0.

Therefore, there exists some function $v : \mathbb{R}_+ \to \mathbb{R}$ such that, for all i and all $\bar{q}_i$,

$$u_i(q(r_0, \bar{q}_i), \bar{q}_i) = v(r_0), \tag{107}$$

for all $r_0 \in \mathbb{R}_+$. Because u_i and q are both continuous and increasing in their first argument, v is also a continuously increasing function.

Finally, let $(q_i, \bar{q}_i) \in \mathbb{R}_+^2$, evaluating the above expression at $r_0 = r(q_i, \bar{q}_i)$, and noticing that

$$q(r(q_i, \bar{q}_i), \bar{q}_i) = q_i \tag{108}$$

yields:

$$u_i(q_i, \bar{q}_i) = v(r(q_i, \bar{q}_i)). \tag{109}$$

This in turn implies that the utility must be a (common) transformation of the responsibility function:

$$u_i = u \equiv v \circ r. \tag{110}$$

Because v is a continuous and increasing function of $\mathbb{R}$, we can write:

$$r = \rho \circ u, \tag{111}$$

with $\rho = v^{-1}$, so that r is a transformation of the common utility function u, as was to be shown. □

We can now proceed with the proof of Theorem 3.

Only if. Let x satisfy **Shared Responsibility** and **EWER**. Because **EWER** is more demanding than **UWUR**, which, in turn, is more demanding than **UWRR**, x must also satisfy **UWRR**. By Theorem 2, x must be a Conditional Equality solution:

$$x_i^{CE}(\mathbf{q},\bar{\mathbf{q}}) = \frac{C(\bar{Q})}{n} + \xi_i\left(\mathbf{r}, C - C(\bar{Q})\right) + u\left(q_i^0, \bar{q}_i\right) - \frac{1}{n}\sum_{j\in N} u\left(q_j^0, \bar{q}_j\right), \tag{112}$$

where u is the common utility function and $\mathbf{q^0}$ is such that, for all $i \in N$, $r\left(q_i^0, \bar{q}_i\right) = r_0$ for some reference responsibility level, r_0.

Because **EWER** is more demanding than **UWUR**, x must also satisfy **UWUR**. By Lemma 1, this can only occur if $u_i = u$ for some utility function u and $r = \rho \circ u$ for some continuous and increasing function ρ. Moreover, it follows from $r = \rho \circ u$ that $u\left(q_i^0, \bar{q}_i\right) = \rho^{-1}\left(r^0\right)$ for all $i \in N$. Hence,

$$x_i^{CE}(\mathbf{q},\bar{\mathbf{q}}) = \frac{C(\bar{Q})}{n} + \xi_i\left(\mathbf{r}, C - C(\bar{Q})\right) \equiv x_i^{UF}(\mathbf{q},\bar{\mathbf{q}}), \qquad \text{for all } i \in N. \tag{113}$$

If. Because x^{UF} coincides with x^{CE} with reference responsibility level $r^0 = 0$ (see Remark 4) we already know from Theorem 2 that x^{UF} satisfies **Shared Responsibility**. Let $(\mathbf{q},\bar{\mathbf{q}}) \in \mathbb{R}_+^{2n}$ such that $r\left(q_i, \bar{q}_i\right) = r\left(q_j, \bar{q}_j\right)$for some $i, j \in N$. It follows from the symmetry of ξ that

$$\xi_i\left(\mathbf{r}, C - C(\bar{Q})\right) = \xi_j\left(\mathbf{r}, C - C(\bar{Q})\right). \tag{114}$$

As a result,

$$x_i^{UF}(\mathbf{q},\bar{\mathbf{q}}) = x_j^{UF}(\mathbf{q},\bar{\mathbf{q}}). \tag{115}$$

Moreover, because $r = \rho \circ u$ for some continuous and increasing function ρ, we can write $u = \rho^{-1} \circ r$. Thus,

$$r\left(q_i, \bar{q}_i\right) = r\left(q_j, \bar{q}_j\right) \implies u\left(q_i, \bar{q}_i\right) = u\left(q_j, \bar{q}_j\right), \tag{116}$$

and $u_i = u_j = u$ yields

$$u_i\left(q_i, \bar{q}_i\right) - x_i^{UF}(\mathbf{q},\bar{\mathbf{q}}) = u_j\left(q_j, \bar{q}_j\right) - x_j^{UF}(\mathbf{q},\bar{\mathbf{q}}). \tag{117}$$

Hence, x^{UF} satisfies **EWER**.

B Section 6 Proofs

B.1 Proof of Proposition 5

Only If. Suppose x satisfies **Solidarity**, **IHR** and **SRRN** with reference needs level $q_0 \geq 0$. Let $(\mathbf{q}, \bar{\mathbf{q}}) \in \mathbb{R}_+^{2n}$ and denote by $\bar{\mathbf{q}}_0 = (\bar{q}_0, \bar{q}_0, ..., \bar{q}_0) \in \mathbb{R}_+^n$ the reference needs vector. By budget balance and anonymity,

$$x_i(\bar{\mathbf{q}}_0, \bar{\mathbf{q}}_0) = \frac{C(n\bar{q}_0)}{n} \qquad \text{for all } i \in N. \tag{118}$$

Without loss of generality, assume that $q_1 \leq q_2 \leq ... \leq q_n$, so that $r_{0,1} \leq r_{0,2} \leq ... \leq r_{0,n}$, where $r_{0,i} = r(q_i, \bar{q}_0)$ for all $i \in N$.
For all $k \in N$, define

$$\mathbf{q}^k = (q_1, q_2, ..., q_{k-1}, q_k, ..., q_k). \tag{119}$$

Notice that $\mathbf{q}^1 = (q_1, q_1, ..., q_1)$. Hence, by anonymity,

$$x_i\left(\mathbf{q}^1, \bar{\mathbf{q}}_0\right) = \frac{C(nq_1)}{n} \tag{120}$$

and

$$x_i\left(\mathbf{q}^1, \bar{\mathbf{q}}_0\right) - x_i(\bar{\mathbf{q}}_0, \bar{\mathbf{q}}_0) = \frac{1}{n}\left[C(nq_1) - C(n\bar{q}_0)\right] \tag{121}$$

for all $i \in N$.

Similarly, for $k \geq 2$, **SRRN** yields

$$x_i\left(\mathbf{q}^k, \bar{\mathbf{q}}_0\right) - x_i(\bar{\mathbf{q}}_0, \bar{\mathbf{q}}_0) = \xi_i\left(\mathbf{r}_0^k, C - C(n\bar{q}_0)\right) \tag{122}$$

and

$$x_i\left(\mathbf{q}^{k-1}, \bar{\mathbf{q}}_0\right) - x_i(\bar{\mathbf{q}}_0, \bar{\mathbf{q}}_0) = \xi_i\left(\mathbf{r}_0^{k-1}, C - C(n\bar{q}_0)\right) \tag{123}$$

for all $i \in N$, with $r_{0,i}^k = r\left(q_i^k, \bar{q}_0\right)$ and $r_{0,i}^{k-1} = r\left(q_i^{k-1}, \bar{q}_0\right)$. Therefore, by subtraction,

$$x_i\left(\mathbf{q}^k, \bar{\mathbf{q}}_0\right) - x_i\left(\mathbf{q}^{k-1}, \bar{\mathbf{q}}_0\right) = \xi_i\left(\mathbf{r}_0^k, C - C(n\bar{q}_0)\right) - \xi_i\left(\mathbf{r}_0^{k-1}, C - C(n\bar{q}_0)\right) \tag{124}$$

for all $i \in N$. Summing up over all consumers, we find:

$$\sum_{i=1}^{n}\left[x_i\left(\mathbf{q}^k, \bar{\mathbf{q}}_0\right) - x_i\left(\mathbf{q}^{k-1}, \bar{\mathbf{q}}_0\right)\right] = C\left(\tilde{Q}^k\right) - C\left(\tilde{Q}^{k-1}\right), \tag{125}$$

where $\tilde{Q}^{k-1} = \sum_{l=1}^{n} q_l^{k-1} = \sum_{l=1}^{k-1} q_l + (n-k+1)\, q_{k-1}$ and $\tilde{Q}^{k} = \sum_{l=1}^{n} q_l^{k} = \sum_{l=1}^{k} q_l + (n-k)\, q_k$.

Observe that if $i < j$ then $r_{0,i}^{k-1} \leq r_{0,j}^{k-1}$ and $r_{0,i}^{k} \leq r_{0,j}^{k}$. Moreover for all $1 \leq i \leq k-1$, $\mathbf{q}_i^{k-1} = \mathbf{q}_i^{k} = q_i$, and $\mathbf{r}_{0,i}^{k-1} = \mathbf{r}_{0,i}^{k} = r\left(q_i, \bar{q}_0\right)$. Therefore, by **IHR**,

$$x_i\left(\mathbf{q}^k, \bar{\mathbf{q}}_0\right) - x_i\left(\mathbf{q}^{k-1}, \bar{\mathbf{q}}_0\right) = 0, \tag{126}$$

for all $1 \leq i \leq k-1$. It follows that the previous summation can truncated from below:

$$\sum_{i=k}^{n} \left[x_i\left(\mathbf{q}^k, \bar{\mathbf{q}}_0\right) - x_i\left(\mathbf{q}^{k-1}, \bar{\mathbf{q}}_0\right)\right] = C\left(\tilde{Q}^k\right) - C\left(\tilde{Q}^{k-1}\right). \tag{127}$$

Moreover, for all $i, j \geq k$, we have $q_i^{k-1} = q_j^{k-1} = q_{k-1}$ and $q_i^k = q_j^k = q_k$. Therefore, by anonymity,

$$x_i\left(\mathbf{q}^{k-1}, \bar{\mathbf{q}}_0\right) = x_j\left(\mathbf{q}^{k-1}, \bar{\mathbf{q}}_0\right) \qquad \text{and} \qquad x_i\left(\mathbf{q}^{k}, \bar{\mathbf{q}}_0\right) = x_j\left(\mathbf{q}^{k}, \bar{\mathbf{q}}_0\right) \tag{128}$$

for all $i, j \geq k$.

Hence,

$$x_i\left(\mathbf{q}^k, \bar{\mathbf{q}}_0\right) - x_i\left(\mathbf{q}^{k-1}, \bar{\mathbf{q}}_0\right) = \frac{1}{n-k+1}\left[C\left(\tilde{Q}^k\right) - C\left(\tilde{Q}^{k-1}\right)\right] \tag{129}$$

for all $i \geq k$, with the convention that $\tilde{Q}^0 = n\bar{q}_0$.

Finally, upon observing that $\mathbf{q}^n = \mathbf{q}$, it follows by summation that

$$x_i\left(\mathbf{q}, \bar{\mathbf{q}}_0\right) - x_i\left(\bar{\mathbf{q}}_0, \bar{\mathbf{q}}_0\right) = \sum_{k=1}^{i} \frac{1}{n-k+1}\left[C\left(\tilde{Q}^k\right) - C\left(\tilde{Q}^{k-1}\right)\right]; \tag{130}$$

i.e., substituting according to Expression (118):

$$x_i\left(\mathbf{q}, \bar{\mathbf{q}}_0\right) = \frac{C(n\bar{q}_0)}{n} + \sum_{k=1}^{i} \frac{1}{n-k+1}\left[C\left(\tilde{Q}^k\right) - C\left(\tilde{Q}^{k-1}\right)\right] \tag{131}$$

We now work along the needs dimension. Define $\bar{\mathbf{q}}_0^1 = \left(\bar{q}_1, \bar{q}_0, ..., \bar{q}_0\right)$. Applying

GS between $(\mathbf{q}, \bar{\mathbf{q}}_0)$ and $(\mathbf{q}, \bar{\mathbf{q}}_0^1)$ yields, for all $j \neq 1$:

$$u(q_1, \bar{q}_1) - x_1^1 - u(q_1, \bar{q}_0) + x_1^0 = u(q_j, \bar{q}_0) - x_j^1 - u(q_j, \bar{q}_0) + x_j^0, \qquad (132)$$

where $x_j^0 = x_j(\mathbf{q}, \bar{\mathbf{q}}_0)$ and $x_j^1 = x_j(\mathbf{q}, \bar{\mathbf{q}}_0^1)$ for all $j \in N$. This yields

$$x_j^0 - x_j^1 = u(q_1, \bar{q}_1) - u(q_1, \bar{q}_0) + x_1^0 - x_1^1. \qquad (133)$$

Since total consumption is unchanged, we have, by budget balance:

$$x_1^1 - x_1^0 = \frac{n-1}{n}\left[u(q_1, \bar{q}_1) - u(q_1, \bar{q}_0)\right], \text{ and} \qquad (134)$$

$$x_j^1 - x_j^0 = -\frac{1}{n}\left[u(q_1, \bar{q}_1) - u(q_1, \bar{q}_0)\right]. \qquad (135)$$

for all $j \neq 1$, (recall the proof of Theorem 1).

Iterating and applying **GS** to profiles $\left(\mathbf{q}, \bar{\mathbf{q}}_0^k\right)$ where $\bar{\mathbf{q}}_0^k = (\bar{q}_1, \bar{q}_2, ..., \bar{q}_k, \bar{q}_0, ..., \bar{q}_0)$, successively leads to the following expression, for all iterations, $k = 1, ..., n$, and all $1 \leq i \leq k \leq j \leq n$:

$$\begin{aligned} u(q_i, \bar{q}_i) - x_i^k - \left[u(q_i, \bar{q}_i) - x_i^{k-1}\right] &= u(q_k, \bar{q}_k) - x_k^k - \left[u(q_k, \bar{q}_0) - x_k^{k-1}\right] \quad (136) \\ &= u(q_j, \bar{q}_0) - x_j^k - \left[u(q_j, \bar{q}_0) - x_j^{k-1}\right] \quad (137) \end{aligned}$$

where $x_j^{k-1} = x_j\left(\mathbf{q}, \bar{\mathbf{q}}_0^{k-1}\right)$ and $x_j^k = x_j\left(\mathbf{q}, \bar{\mathbf{q}}_0^k\right)$. Hence, for all $k = 1, ..., n$, and all $1 \leq i \leq k \leq j \leq n$:

$$\begin{aligned} x_i^{k-1} - x_i^k &= u(q_k, \bar{q}_k) - u(q_k, \bar{q}_0) + x_k^{k-1} - x_k^k \quad (138) \\ &= x_j^{k-1} - x_j^k \quad (139) \end{aligned}$$

Since total consumption does not change from $\left(\mathbf{q}, \bar{\mathbf{q}}_0^{k-1}\right)$ to $\left(\mathbf{q}, \bar{\mathbf{q}}_0^k\right)$, but only needs, budget balance implies $\sum_j \left(x_j^k - x_j^{k-1}\right) = 0$. Therefore,

$$x_j^k - x_j^{k-1} = -\frac{1}{n}\left[u(q_k, \bar{q}_k) - u(q_k, \bar{q}_0)\right] \qquad \text{for all } j \neq k \text{, and} \qquad (140)$$

$$x_k^k - x_k^{k-1} = \frac{n-1}{n}\left[u(q_k, \bar{q}_k) - u(q_k, \bar{q}_0)\right] \qquad (141)$$

Summing up over all iterations $k = 1, ..., n$ yields the following for consumer 1:

$$x_1^n - x_1^0 = \sum_{k>1}^{n} \left(x_1^k - x_1^{k-1} \right) + x_1^1 - x_1^0 \tag{142}$$

$$= -\frac{1}{n} \sum_{k>1}^{n} \left[u \left(q_k, \bar{q}_k \right) - u \left(q_k, \bar{q}_0 \right) \right] + \frac{n-1}{n} \left[u \left(q_1, \bar{q}_1 \right) - u \left(q_1, \bar{q}_0 \right) \right] \tag{143}$$

$$= \left[u \left(q_1, \bar{q}_1 \right) - u \left(q_1, \bar{q}_0 \right) \right] - \frac{1}{n} \sum_{k=1}^{n} \left[u \left(q_k, \bar{q}_k \right) - u \left(q_k, \bar{q}_0 \right) \right] \tag{144}$$

Similarly, for all $i > 1$:

$$x_i^n - x_i^0 = \left[u \left(q_i, \bar{q}_i \right) - u \left(q_i, \bar{q}_0 \right) \right] - \frac{1}{n} \sum_{k=1}^{n} \left[u \left(q_k, \bar{q}_k \right) - u \left(q_k, \bar{q}_0 \right) \right] \tag{145}$$

Finally, observing that $\bar{\mathbf{q}}_0^n = \bar{\mathbf{q}}$, Expressions (131) and (145) yield the following :

$$x_i \left(\mathbf{q}, \bar{\mathbf{q}} \right) = x_i \left(\mathbf{q}, \bar{\mathbf{q}}_0^n \right) = \frac{C(n\bar{q}_0)}{n} + \sum_{k=1}^{i} \frac{1}{n-k+1} \left[C \left(\tilde{Q}^k \right) - C \left(\tilde{Q}^{k-1} \right) \right] \tag{146}$$

$$+ \left[u \left(q_i, \bar{q}_i \right) - u \left(q_i, \bar{q}_0 \right) \right] - \frac{1}{n} \sum_{k=1}^{n} \left[u \left(q_k, \bar{q}_k \right) - u \left(q_k, \bar{q}_0 \right) \right]$$

for all $i \in N$, where $\tilde{Q}^k = \sum_{l=1}^{k} q_l + (n-k) q_k$ for all $k = 1, ..., n$.

If. By construction, the above solution satisfies **Solidarity**, **IHR** and **SRRN** with reference needs level $q_0 \geq 0$.

B.2 Proof of Proposition

Only If. Suppose x satisfies **Shared Responsibility**, **IHR** and **UWRR** with reference responsibility level $r_0 \geq 0$. Let $(\mathbf{q}, \bar{\mathbf{q}}) \in \mathbb{R}_+^{2n}$ and, without loss of generality, assume that $r_1 \leq r_2 \leq ... \leq r_n$. Define $\mathbf{q^0} \in \mathbb{R}_+^n$ such that,

$$r \left(q_i^0, \bar{q}_i \right) = r_0, \qquad \text{for all } i \in N. \tag{147}$$

By **UWRR**,

$$u_i \left(q_i^0, \bar{q}_i \right) - x_i \left(\mathbf{q}^0, \bar{\mathbf{q}} \right) = u_j \left(q_j^0, \bar{q}_j \right) - x_j \left(\mathbf{q}^0, \bar{\mathbf{q}} \right), \qquad \text{for all } i, j \in N. \tag{148}$$

Hence, for all $i \in N$,

$$x_i\left(\mathbf{q}^0, \bar{\mathbf{q}}\right) = u_i\left(q_i^0, \bar{q}_i\right) - \frac{1}{n}\sum_{j \in N}\left[u_j\left(q_j^0, \bar{q}_j\right) - x_j\left(\mathbf{q}^0, \bar{\mathbf{q}}\right)\right], \tag{149}$$

$$= \frac{1}{n}C\left(Q^0\right) + u_i\left(q_i^0, \bar{q}_i\right) - \frac{1}{n}\sum_{j \in N} u_j\left(q_j^0, \bar{q}_j\right), \tag{150}$$

by budget balance, where $Q^0 \equiv \sum_{j \in N} q_j^0$.

Applying **Shared Responsibility** between profiles $(\mathbf{q}^0, \bar{\mathbf{q}})$ and $(\bar{\mathbf{q}}, \bar{\mathbf{q}})$ yields:

$$x_i\left(\mathbf{q}^0, \bar{\mathbf{q}}\right) - x_i\left(\bar{\mathbf{q}}, \bar{\mathbf{q}}\right) \quad = \quad \xi_i\left(\mathbf{r}_0, C - C(\bar{Q})\right). \tag{151}$$

Applying now **Shared Responsibility** between profiles $(\bar{\mathbf{q}}, \bar{\mathbf{q}})$ and $(\mathbf{q}, \bar{\mathbf{q}})$ yields:

$$x_i\left(\mathbf{q}, \bar{\mathbf{q}}\right) - x_i\left(\bar{\mathbf{q}}, \bar{\mathbf{q}}\right) \quad = \quad \xi_i\left(\mathbf{r}, C - C(\bar{Q})\right). \tag{152}$$

Combining expressions (151) and (152) yields:

$$x_i\left(\mathbf{q}, \bar{\mathbf{q}}\right) - x_i\left(\mathbf{q}^0, \bar{\mathbf{q}}\right) = \xi_i\left(\mathbf{r}, C - C(\bar{Q})\right) - \xi_i\left(\mathbf{r}_0, C - C(\bar{Q})\right) \tag{153}$$

Let $f_i : w \mapsto r\left(w, \bar{q}_i\right)$ map consumption to individual responsibility for consumer i. By construction, f_i is strictly increasing. Its inverse, $g_i : v \mapsto f_i^{-1}(v)$, is well defined and is also strictly increasing. In particular, $g_i\left(r_i\right) = q_i$ for all $i \in N$.

Define the following profile:

$$\mathbf{q}^1 = \left(q_1, g_2\left(r_1\right), ..., g_i\left(r_1\right), ..., g_n\left(r_1\right)\right). \tag{154}$$

Note that, by construction, $(\mathbf{q}^1, \bar{\mathbf{q}})$ is such that $r_i^1 = r_1$ for all $i \in N$. By symmetry of ξ and because $r_i^1 = r_1$ for all $i \in N$, we have:

$$\xi_i\left(\mathbf{r}^1, C - C\left(\bar{Q}\right)\right) = \frac{1}{n}\left[C\left(\hat{Q}^1\right) - C\left(\bar{Q}\right)\right], \tag{155}$$

where

$$\hat{Q}^1 = \sum_{i=1}^{n} q_i^1 = \sum_{i=1}^{n} g_i\left(r_1\right). \tag{156}$$

Similarly, let

$$\mathbf{q}^2 = \left(q_1, g_2\left(r_2\right), g_3\left(r_2\right), ..., g_i\left(r_2\right), ..., g_n\left(r_2\right)\right). \tag{157}$$

Again, by construction $(\mathbf{q}^2, \bar{\mathbf{q}})$ is such that $r_i^2 = r_2$ for all $i = 2, ..., n$. Because $r_1 \leq r_2$, applying **IHR** between profiles $(\mathbf{q}^1, \bar{\mathbf{q}})$ and $(\mathbf{q}^2, \bar{\mathbf{q}})$ yields that consumer 1's contribution is the same under both profiles:

$$\xi_1\left(\mathbf{r}^2, C - C\left(\bar{Q}\right)\right) = \xi_1\left(\mathbf{r}^1, C - C\left(\bar{Q}\right)\right), \tag{158}$$

$$= \frac{1}{n}\left[C\left(\hat{Q}^1\right) - C\left(\bar{Q}\right)\right] \tag{159}$$

Thus, consumers $2, ..., n$ share the remaining cost equally:

$$\xi_i\left(\mathbf{r}^2, C - C\left(\bar{Q}\right)\right) - \xi_i\left(\mathbf{r}^1, C - C\left(\bar{Q}\right)\right) = \frac{1}{n-1}\left[C\left(\hat{Q}^2\right) - C\left(\hat{Q}^1\right)\right], \tag{160}$$

where

$$\hat{Q}^2 = \sum_{j=1}^{n} q_j^2 = q_1 + \sum_{j=2}^{n} g_j\left(r_2\right) \geq \hat{Q}^1. \tag{161}$$

More generally, let $k \in \{2, ..., n\}$ and define

$$\mathbf{q}^k = \left(q_1, q_2, ..., q_{k-1}, g_k\left(r_k\right), ..., g_n\left(r_k\right)\right). \tag{162}$$

It follows from **IHR** and symmetry of ξ that

$$\begin{cases} \xi_i\left(\mathbf{r}^k, C - C\left(\bar{Q}\right)\right) - \xi_i\left(\mathbf{r}^{k-1}, C - C\left(\bar{Q}\right)\right) = 0 & \text{for all } i < k \text{, and} \\ \xi_i\left(\mathbf{r}^k, C - C\left(\bar{Q}\right)\right) - \xi_i\left(\mathbf{r}^{k-1}, C - C\left(\bar{Q}\right)\right) = \frac{1}{n-k+1}\left[C\left(\hat{Q}^k\right) - C\left(\hat{Q}^{k-1}\right)\right] & \text{for all } i \geq k, \end{cases} \tag{163}$$

with

$$\hat{Q}^k = \sum_{j=1}^{n} q_j^k = \sum_{j=1}^{k-1} q_j + \sum_{j=k}^{n} g_j\left(r_k\right). \tag{164}$$

Summing up over all iterations, we obtain for all $i \in \{1, ..., n\}$:

$$\xi_i\left(\mathbf{r}, C - C(\bar{Q})\right) - \xi_i\left(\mathbf{r}_1, C - C(\bar{Q})\right) = \sum_{k=2}^{i}\left[\xi_i\left(\mathbf{r}^k, C - C\left(\bar{Q}\right)\right) - \xi_i\left(\mathbf{r}^{k-1}, C - C\left(\bar{Q}\right)\right)\right] \tag{165}$$

$$= \sum_{k=2}^{i}\frac{1}{n-k+1}\left[C\left(\hat{Q}^k\right) - C\left(\hat{Q}^{k-1}\right)\right]. \tag{166}$$

so that, recalling Expression (155),

$$\xi_i\left(\mathbf{r}, C - C(\bar{Q})\right) = \frac{1}{n}\left[C\left(\hat{Q}^1\right) - C\left(\bar{Q}\right)\right] + \sum_{k=2}^{i}\frac{1}{n-k+1}\left[C\left(\hat{Q}^k\right) - C\left(\hat{Q}^{k-1}\right)\right]. \tag{167}$$

Putting everything together, and recalling Expression (152), the above expression becomes

$$x_i\left(\mathbf{q}, \bar{\mathbf{q}}\right) = \frac{1}{n}\left[C\left(\hat{Q}^1\right) - C\left(\bar{Q}\right)\right] + \sum_{k=2}^{i}\frac{1}{n-k+1}\left[C\left(\hat{Q}^k\right) - C\left(\hat{Q}^{k-1}\right)\right] \tag{168}$$

$$+ x_i\left(\bar{\mathbf{q}}, \bar{\mathbf{q}}\right) \tag{169}$$

$$= \frac{1}{n}\left[C\left(\hat{Q}^1\right) - C\left(\bar{Q}\right)\right] + \sum_{k=2}^{i}\frac{1}{n-k+1}\left[C\left(\hat{Q}^k\right) - C\left(\hat{Q}^{k-1}\right)\right] \tag{170}$$

$$- \frac{1}{n}\left[C\left(Q^0\right) - C\left(\bar{Q}\right)\right] + x_i\left(\mathbf{q}^0, \bar{\mathbf{q}}\right) \tag{171}$$

$$= \frac{1}{n}\left[C\left(\hat{Q}^1\right) - C\left(\bar{Q}\right)\right] + \sum_{k=2}^{i}\frac{1}{n-k+1}\left[C\left(\hat{Q}^k\right) - C\left(\hat{Q}^{k-1}\right)\right] \tag{172}$$

$$- \frac{1}{n}\left[C\left(Q^0\right) - C\left(\bar{Q}\right)\right] + \frac{1}{n}C\left(Q^0\right) + u_i\left(q_i^0, \bar{q}_i\right) - \frac{1}{n}\sum_{j\in N}u_j\left(q_j^0, \bar{q}_j\right) \tag{173}$$

$$= \frac{1}{n}C\left(\hat{Q}^1\right) + \sum_{k=2}^{i}\frac{1}{n-k+1}\left[C\left(\hat{Q}^k\right) - C\left(\hat{Q}^{k-1}\right)\right] \tag{174}$$

$$+ u_i\left(q_i^0, \bar{q}_i\right) - \frac{1}{n}\sum_{j\in N}u_j\left(q_j^0, \bar{q}_j\right) \tag{175}$$

where the second and third equalities come from Expressions (151) and (150), respectively.

If. By construction, the above solution satisfies **Shared Responsibility**, **IHR** and **UWRR** with reference responsibility level $r_0 \geq 0$.

B.3 Proof of Proposition 7

The proof follows the structure of that of Theorem 3.

Only If. Suppose x satisfies **Shared Responsibility**, **EWER** and **IHR**. Because **EWER** is more demanding than **UWRR**, Proposition 6 applies, so that x must be a Serial Conditional Equality solution:

$$x_i(\mathbf{q}, \bar{\mathbf{q}}) = \frac{1}{n} C\left(\hat{Q}^1\right) + \sum_{k=2}^{i} \frac{1}{n-k+1} \left[C\left(\hat{Q}^i\right) - C\left(\hat{Q}^{i-1}\right)\right] \tag{176}$$

$$+ u_i\left(q_i^0, \bar{q}_i\right) - \frac{1}{n} \sum_{j \in N} u_j\left(q_j^0, \bar{q}_j\right) \tag{177}$$

for some reference level of responsibility, $r_0 \geq 0$.

Because **EWER** implies **UWUR**, Lemma 1 applies. Hence, there must exist some common utility function, u, such that $u_i = u$ for all i, and the responsibility function must reflect this common utility function: $r = \rho \circ u$ for some continuous and increasing function ρ. Moreover, it follows from $r = \rho \circ u$ that $u\left(q_i^0, \bar{q}_i\right) = \rho^{-1}\left(r^0\right)$ for all $i \in N$.

Hence,

$$x_i(\mathbf{q}, \bar{\mathbf{q}}) = \frac{1}{n} C\left(\hat{Q}^1\right) + \sum_{k=2}^{i} \frac{1}{n-k+1} \left[C\left(\hat{Q}^i\right) - C\left(\hat{Q}^{i-1}\right)\right] \tag{178}$$

If. We already know from Theorem 6 that x^{SUF} satisfies **Shared Responsibility**. Let $(\mathbf{q}, \bar{\mathbf{q}}) \in \mathbb{R}_+^{2n}$ such that $r(q_i, \bar{q}_i) = r(q_j, \bar{q}_j)$for some $i, j \in N$. It follows from the symmetry of ξ that

$$\xi_i\left(\mathbf{r}, C - C(\bar{Q})\right) = \xi_j\left(\mathbf{r}, C - C(\bar{Q})\right). \tag{179}$$

As a result,

$$x_i^{SUF}(\mathbf{q}, \bar{\mathbf{q}}) = x_j^{SUF}(\mathbf{q}, \bar{\mathbf{q}}). \tag{180}$$

Moreover, because $r = \rho \circ u$ for some continuous and increasing function ρ, we can write $u = \rho^{-1} \circ r$. Thus,

$$r\left(q_i, \bar{q}_i\right) = r\left(q_j, \bar{q}_j\right) \implies u\left(q_i, \bar{q}_i\right) = u\left(q_j, \bar{q}_j\right), \tag{181}$$

and $u_i = u_j = u$ yields

$$u_i\left(q_i, \bar{q}_i\right) - x_i^{SUF}\left(\mathbf{q}, \bar{\mathbf{q}}\right) = u_j\left(q_j, \bar{q}_j\right) - x_j^{SUF}\left(\mathbf{q}, \bar{\mathbf{q}}\right). \tag{182}$$

Hence, x^{SUF} satisfies **EWER**.

C Supplementary material: Calculations not intended for publication

For the upcoming calculations, recall the notations of Section 7.

C.1 Calculations of rate functions with a population distribution

SEE

Recall the expression of x^{SEE} in the discrete setting. Fix a reference level of needs, $\bar{q}_0$. The corresponding x^{SEE} rate function is, for all $i \in N$:

$$x_i^{EE}(\mathbf{q}, \bar{\mathbf{q}}) = \frac{C(n\bar{q}_0)}{n} + \sum_{k=1}^{i} \frac{1}{n-k+1} \left[C\left(\tilde{Q}^k\right) - C\left(\tilde{Q}^{k-1}\right) \right] \tag{183}$$

$$+ \left[u_i(q_i, \bar{q}_i) - u_i(q_i, \bar{q}_0) \right] - \frac{1}{n} \sum_{k=1}^{n} \left[u_k(q_k, \bar{q}_k) - u_k(q_k, \bar{q}_0) \right] \tag{184}$$

where $\tilde{Q}^k = \sum_{l=1}^{k} q_l + (n-k) q_k$ with the set of consumers ordered so as to have $q_1 \leq q_2 \leq ... \leq q_n$.

When representing the population with distributions, one obtains that, for any given consumption level q,

$$\tilde{Q}(q) = \int_0^\infty \inf\{q, z\} n(z)\, dz. \tag{185}$$

Proceeding term for term, the translation from the discrete setting to the distributional one is rather straightforward. The only term that requires a few steps is the second term of (183), which amounts to

$$\int_{z=0}^{q} \frac{1}{N - N(q)} C'\left(\tilde{Q}(z)\right) \frac{d\tilde{Q}(z)}{dz} dz. \tag{186}$$

Notice that $d\tilde{Q}(q) = [N - N(q)]\, dq$ (because $\tilde{Q}(q)$ increases by dq for every consumer who consumes q or more. It follows that (186) simplifies into

$$\int_{z=0}^{q} C'\left(\tilde{Q}\left(z\right)\right) dz,$$

as was to be shown.

SCE

Recall the expression of x^{SCE} in the discrete setting. Fix a reference responsibility level, r_0. The corresponding x^{SCE} rate function is, for all $i \in N$:

$$x_i^{SCE}\left(\mathbf{q}, \bar{\mathbf{q}}\right) = \frac{1}{n} C\left(\hat{Q}^0\right) + \sum_{k=1}^{i} \frac{1}{n-k}\left[C\left(\hat{Q}^i\right) - C\left(\hat{Q}^{i-1}\right)\right] \quad (187)$$

$$+u_i\left(q_i^0, \bar{q}_i\right) - \frac{1}{n}\sum_{j\in N} u_j\left(q_j^0, \bar{q}_j\right) \quad (188)$$

where, $\hat{Q}^0 \equiv \sum_{j\in N} q_j^0$ and, for all $k \in N$, $\hat{Q}^k = \sum_{j=1}^{k} q_j + \sum_{j=k-1}^{n} g_j\left(r_k\right)$, with the set of consumers is ordered so as to have $r_1 \leq r_2 \leq ... \leq r_n$.

When representing the population with distributions, one obtains that, for any given responsibility level ρ,

$$\hat{Q}\left(\rho\right) = \sum_{s\in S}\left[\int_{0}^{+\infty} g_s\left(\inf\{\rho, z\}\right) n_s^r\left(z\right) dz\right]. \quad (189)$$

Proceeding term for term, the translation from the discrete setting to the distributional one is rather straightforward. The only term that requires a few steps is the second term of (187), which amounts to

$$\int_{z=0}^{\rho} \frac{1}{N - N^r\left(z\right)} C'\left(\hat{Q}\left(z\right)\right) \frac{d\hat{Q}\left(z\right)}{dz} dz. \quad (190)$$

Notice that $d\hat{Q}\left(\rho\right) = \sum_{s\in S}\left[N_s - N_s^r\left(\rho\right)\right]\left(dg_s\left(\rho\right)/d\rho\right) d\rho$ because a variation in responsbility, $d\rho$, corresponds to a variation in consumption, $dg_s\left(\rho\right)/d\rho$, for every consumer whose responsbility is ρ or more.

C.2 Decreasing Returns to Scale: Quadratic Costs

SUF with absolute responsibility

Recall that

$$x^{SUF}(\rho)=\frac{C\left(\bar{Q}\right)}{N}+\int_{z=0}^{\rho}\frac{1}{N-N^{r}(z)}C'\left(\hat{Q}(z)\right)\frac{d\hat{Q}(z)}{d\rho}dz, \tag{191}$$

where

$$\hat{Q}(\rho)=\sum_{s\in S}\left[\int_{0}^{+\infty}\inf\{g_s(z),g_s(\rho)\}n_s^r(z)\,dz\right] \tag{192}$$

$$=\sum_{s\in S}\left[\int_{0}^{\rho}g_s(z)\,n_s^r(z)\,dz+g_s(\rho)\int_{\rho}^{+\infty}n_s^r(z)\,dz\right] \tag{193}$$

Under the absolute responsibility view,

$$\frac{d\hat{Q}(\rho)}{d\rho}=\sum_{s\in S}\left\{g_s(\rho)\,n_s^r(\rho)-g_s(\rho)\,n_s^r(\rho)+g_s'(\rho)\int_{\rho}^{+\infty}n_s^r(z)\,dz\right\},$$
$$=\sum_{s\in S}g_s'(\rho)\left[N_s-N_s^r(\rho)\right]$$

$$\frac{d\hat{Q}(\rho)}{d\rho}=\sum_{s\in S}\left(N_s-N_s^r(\rho)\right)g_s'(\rho)=N-N^r(\rho), \tag{194}$$

with the second equality following from the fact that $g_s(\rho)\equiv\bar{q}_s+\rho$. Hence,

$$x^{SUF}(\rho)=\frac{C\left(\bar{Q}\right)}{N}+\int_{z=0}^{\rho}C'\left(\hat{Q}(z)\right)dz, \tag{195}$$

with

$$\hat{Q}_s(\rho)=\int_{0}^{\rho}(\bar{q}_s+z)\,n_s^r(z)\,dz+\left(N_s-N_s^r(\rho)\right)(\bar{q}_s+\rho) \tag{196}$$

$$=\bar{q}_sN_s+\int_{0}^{\rho}zn_s^r(z)\,dz+\left(N_s-N_s^r(\rho)\right)\rho \tag{197}$$

$$=\bar{q}_sN_s+\int_{0}^{+\infty}\min\{z,\rho\}n_s^r(z)\,dz, \tag{198}$$

so that

$$\hat{Q}(\rho) = \bar{Q} + \int_0^{+\infty} \min\{z,\rho\} n^r(z)\, dz \tag{199}$$

$$= \bar{Q} + \int_0^{\rho} z n^r(z)\, dz + (N - N^r(\rho))\rho \tag{200}$$

Consider the case where $C(Q) = \frac{c}{2}Q^2$. It follows that $C'(Q) = cQ$, so that

$$x^{SUF}(\rho) = \frac{c\bar{Q}^2}{2N} + c\int_{z=0}^{\rho} \hat{Q}(z)\, dz \tag{201}$$

$$= \frac{c\bar{Q}^2}{2N} + c\int_{z=0}^{\rho} \left[\bar{Q} + \int_{y=0}^{+\infty} \min\{y,z\} n^r(y)\, dy\right] dz \tag{202}$$

$$= \frac{c\bar{Q}^2}{2N} + c\bar{Q}\rho + c\int_{y=0}^{+\infty} n^r(y) \int_{z=0}^{\rho} \min\{y,z\} dz dy \tag{203}$$

$$= \frac{c\bar{Q}^2}{2N} + c\bar{Q}\rho + c\int_{y=0}^{+\infty} n^r(y) \left[\int_{z=0}^{y} z dz + \int_{z=y}^{\rho} y dz\right] dy \tag{204}$$

$$= \frac{c\bar{Q}^2}{2N} + c\bar{Q}\rho + c\int_{y=0}^{+\infty} n^r(y) \left[\frac{y^2}{2} + y(\rho - y)\right] dy, \tag{205}$$

$$= \frac{c\bar{Q}^2}{2N} + c\bar{Q}\rho + c\int_{y=0}^{+\infty} n^r(y) \left[y\rho - \frac{y^2}{2}\right] dy. \tag{206}$$

Upon noticing that, under absolute responsibility, the sum total of the population's responsibility writes $\int_{y=0}^{+\infty} n^r(y)\, y dy = Q - \bar{Q}$, the above expression rewrites as follows:

$$x^{SUF}(\rho) = \frac{c\bar{Q}^2}{2N} - c\int_{y=0}^{+\infty} n^r(y) \frac{y^2}{2} dy + cQ\rho. \tag{207}$$

By budget balance,

$$c\frac{Q^2}{2} = \int_{z=0}^{+\infty} x^{SUF}(z)\, n^r(z)\, dz \tag{208}$$

$$= N\left[\frac{c\bar{Q}^2}{2N} - c\int_{y=0}^{+\infty} n^r(y) \frac{y^2}{2} dy\right] + cQ\int_{z=0}^{+\infty} z n^r(z)\, dz \tag{209}$$

$$= N\left[\frac{c\bar{Q}^2}{2N} - c\int_{y=0}^{+\infty} n^r(y) \frac{y^2}{2} dy\right] + cQ\left(Q - \bar{Q}\right). \tag{210}$$

Thus,

$$\frac{c\bar{Q}^2}{2N} - c\int_{y=0}^{+\infty} n^r(y)\frac{y^2}{2}dy = \frac{1}{N}\left(\frac{cQ^2}{2} - cQ\left(Q-\bar{Q}\right)\right). \tag{211}$$

Finally, it follows that

$$x^{SUF}(\rho) = \frac{1}{N}\frac{cQ^2}{2} + cQ\left(\rho - \frac{Q-\bar{Q}}{N}\right) \tag{212}$$

Upon recalling that $\rho = q - \bar{q}_s$ under absolute responsibility, we obtain the result:

$$x^{SUF}(q,s) = \frac{1}{N}\frac{cQ^2}{2} + cQ\left(q - \bar{q}_s - \frac{Q-\bar{Q}}{N}\right). \tag{213}$$

SUF with relative responsibility

Recall that

$$x^{SUF}(\rho) = \frac{C\left(\bar{Q}\right)}{N} + \int_{z=0}^{\rho} \frac{1}{N - N^r(z)} C'\left(\hat{Q}(z)\right)\frac{d\hat{Q}(z)}{dz}dz, \tag{214}$$

where, from Expressions (199) and (200), we can write:

$$\begin{aligned}\hat{Q}(\rho) &= \sum_{s\in S}\left[\int_0^{+\infty} g_s(\inf\{z,\rho\})\, n_s^r(z)\, dz\right] &(215)\\ &= \sum_{s\in S}\left[\int_0^{\rho} g_s(z)\, n_s^r(z)\, dz + (N_s - N_s^r(\rho))\, g_s(\rho)\right] &(216)\end{aligned}$$

Under relative responsibility, $\rho = (q-\bar{q}_s)/\bar{q}_s$ so that $g_s(\rho) = \bar{q}_s(1+\rho)$. It follows that $g_s'(\rho) = \bar{q}_s$ and

$$\frac{d\hat{Q}_s(\rho)}{d\rho} = (N_s - N_s^r(\rho))\, g_s'(\rho) = (N_s - N_s^r(\rho))\, \bar{q}_s. \tag{217}$$

We now make an additional assumption. Namely, we posit that responsibility is evenly spread across types, so that its distribution is independent of needs, $\bar{q}_s$:

$$N_s^r(\rho) = \alpha(\rho)\, N_s \qquad \forall s \in S, \tag{218}$$

for some increasing function $\alpha : \mathbb{R}_+ \to [0,1]$ which we take to be differentiable. This yields:

$$\frac{d\hat{Q}(\rho)}{d\rho} = (1-\alpha(\rho))\bar{Q}. \tag{219}$$

Also, because $N - N^r(\rho) = (1-\alpha(\rho))N$, we have

$$\frac{1}{N-N^r(\rho)}\frac{d\hat{Q}(\rho)}{d\rho} = \frac{\bar{Q}}{N}, \tag{220}$$

so that $x^{SUF}(\rho)$ simplifies to

$$x^{SUF}(\rho) = \frac{C(\bar{Q})}{N} + \frac{\bar{Q}}{N}\int_{z=0}^{\rho} C'\left(\hat{Q}(z)\right) dz. \tag{221}$$

Upon noticing that $n_s^r(\rho) = \alpha'(\rho) N_s$ we get

$$\hat{Q}(\rho) = \int_0^{+\infty} \sum_{s\in S} \inf\{g_s(z), g_s(\rho)\} N_s \alpha'(z)\, dz \tag{222}$$

$$= \int_0^{+\infty} \inf\{\sum_{s\in S} N_s g_s(z), \sum_{s\in S} N_s g_s(\rho)\}\alpha'(z)\, dz \tag{223}$$

where the summation sign enters the minimum operator because, for any $s \in S$, $g_s(z) \leq g_s(\rho)$ if and only if $z \leq \rho$. Therefore,

$$\hat{Q}(\rho) = \int_0^{+\infty} \inf\{\sum_{s\in S} N_s \bar{q}_s(1+z), \sum_{s\in S} N_s \bar{q}_s(1+\rho)\}\alpha'(z)\, dz \tag{224}$$

$$= \bar{Q}\left[1 + \int_0^{+\infty} \inf\{z,\rho\}\alpha'(z)\, dz\right]. \tag{225}$$

Assuming $C(Q) = \frac{1}{2}cQ^2$,

$$
\begin{aligned}
x^{SUF}(\rho) &= \frac{c\bar{Q}^2}{2N} + \frac{\bar{Q}c}{N}\int_{z=0}^{\rho} \hat{Q}(z)\,dz \\
&= \frac{c\bar{Q}^2}{2N} + \frac{\bar{Q}c}{N}\int_{z=0}^{\rho} \bar{Q}\left[1 + \int_{y=0}^{+\infty} \inf\{y,z\}\alpha'(y)\,dy\right] dz \\
&= \frac{c\bar{Q}^2}{2N} + \frac{\bar{Q}^2 c\rho}{N} + \frac{\bar{Q}^2 c}{N}\int_{y=0}^{+\infty}\int_{z=0}^{r} \inf\{y,z\}\alpha'(y)\,dydz \\
&= \frac{c\bar{Q}^2}{2N} + \frac{\bar{Q}^2 c\rho}{N} + \frac{\bar{Q}^2 c}{N}\int_{y=0}^{+\infty} \alpha'(y)\left[\int_{z=0}^{y} zdz + y\int_{z=y}^{\rho} dz\right] dy \\
&= \frac{c\bar{Q}^2}{2N} + \frac{\bar{Q}^2 c\rho}{N} + \frac{\bar{Q}^2 c}{N}\int_{y=0}^{+\infty} \frac{n^r(y)}{N}\left[\frac{y^2}{2} + y(\rho - y)\right] dy \\
&= \frac{c\bar{Q}^2}{2N} + \frac{c\bar{Q}^2}{N}\rho + \frac{c\bar{Q}^2}{N^2}\int_{y=0}^{+\infty} \left[\left(\rho - \frac{y}{2}\right) yn^r(y)\right] dy \\
&= \frac{c\bar{Q}^2}{2N} - \frac{c\bar{Q}^2}{2N^2}\int_{y=0}^{+\infty} y^2 n^r(y)\,dy + \frac{c\bar{Q}^2}{N}\left[1 + \frac{1}{N}\int_{y=0}^{+\infty} yn^r(y)\,dy\right]\rho.
\end{aligned}
$$

For households of type s this writes:

$$
\begin{aligned}
x^{SUF}(q,s) &= \frac{c\bar{Q}^2}{2N} - \frac{c\bar{Q}^2}{2N^2}\int_{y=0}^{+\infty} y^2 n^r(y)\,dy + \frac{c\bar{Q}^2}{N}\left[1 + \frac{1}{N}\int_{y=0}^{+\infty} yn^r(y)\,dy\right]\left(\frac{q - \bar{q}_s}{\bar{q}_s}\right) \\
&= \left\{\frac{c\bar{Q}^2}{2N} - \frac{c\bar{Q}^2}{2N^2}\int_{y=0}^{+\infty} y^2 n^r(y)\,dy - \frac{c\bar{Q}^2}{N}\left[1 + \frac{1}{N}\int_{y=0}^{+\infty} yn^r(y)\,dy\right]\right\} \\
&\quad + \frac{c\bar{Q}^2}{N}\left[1 + \frac{1}{N}\int_{y=0}^{+\infty} yn^r(y)\,dy\right]\frac{q}{\bar{q}_s}.
\end{aligned}
$$

Also, by budget balance,

$$\frac{cQ^2}{2} = \sum_s \int_{z=0}^{+\infty} x^{SUF}(z)\, n_s^r(z)\, dz \tag{226}$$

$$= \sum_s \int_{z=0}^{+\infty} \left\{ \frac{c\bar{Q}^2}{2N} - \frac{c\bar{Q}^2}{2N^2} \int_{y=0}^{+\infty} y^2 n^r(y)\, dy - \frac{c\bar{Q}^2}{N} \left[1 + \frac{1}{N} \int_{y=0}^{+\infty} y n^r(y)\, dy \right] \right\} n_s^r(z)\, dz \tag{227}$$

$$+ \sum_s \int_z \frac{c\bar{Q}^2}{N} \left[1 + \frac{1}{N} \int_{y=0}^{+\infty} y n^r(y)\, dy \right] \frac{q}{\bar{q}_s} n_s^r(z)\, dz$$

$$= \left\{ \frac{c\bar{Q}^2}{2N} - \frac{c\bar{Q}^2}{2N^2} \int_{y=0}^{+\infty} y^2 n^r(y)\, dy - \frac{c\bar{Q}^2}{N} \left[1 + \frac{1}{N} \int_{y=0}^{+\infty} y n^r(y)\, dy \right] \right\} \sum_s \int_{z=0}^{+\infty} n_s^r(z)\, dz \tag{228}$$

$$+ \frac{c\bar{Q}^2}{N} \left[1 + \frac{1}{N} \int_{y=0}^{+\infty} y n^r(y)\, dy \right] \sum_s \int_z (z+1)\, n_s^r(z)\, dz$$

$$= \left\{ \frac{c\bar{Q}^2}{2N} - \frac{c\bar{Q}^2}{2N^2} \int_{y=0}^{+\infty} y^2 n^r(y)\, dy - \frac{c\bar{Q}^2}{N} \left[1 + \frac{1}{N} \int_{y=0}^{+\infty} y n^r(y)\, dy \right] \right\} N \tag{229}$$

$$+ \frac{c\bar{Q}^2}{N} \left[1 + \frac{1}{N} \int_{y=0}^{+\infty} y n^r(y)\, dy \right] \left[\sum_s \frac{Q_s}{\bar{q}_s} \right].$$

because $z+1 = g_s(z)/\bar{q}_s$ and $\int_z (z+1)\, n_s^r(z)\, dz = \int_z [g_s(z)/\bar{q}_s]\, n_s^r(z)\, dz = \int_q (q/\bar{q}_s)\, n_s(q)\, dq = Q_s/\bar{q}_s$.

Therefore,

$$\left\{ \frac{c\bar{Q}^2}{2N} - \frac{c\bar{Q}^2}{2N^2} \int_{y=0}^{+\infty} y^2 n^r(y)\, dy - \frac{c\bar{Q}^2}{N} \left[1 + \frac{1}{N} \int_{y=0}^{+\infty} y n^r(y)\, dy \right] \right\} = \frac{1}{N} \left\{ \frac{cQ^2}{2} \right. \tag{230}$$

$$- \frac{c\bar{Q}^2}{N} \left[1 + \frac{1}{N} \int_{y=0}^{+\infty} y n^r(y)\, dy \right] \left[\sum_s \right.$$

Hence,

$$x^{SUF}(q,s) = \frac{1}{N} \left\{ \frac{cQ^2}{2} - \frac{c\bar{Q}^2}{N} \left[1 + \frac{1}{N} \int_{y=0}^{+\infty} y n^r(y)\, dy \right] \left[\sum_s \frac{Q_s}{\bar{q}_s} \right] \right\} + \frac{c\bar{Q}^2}{N} \left[1 + \frac{1}{N} \int_{y=0}^{+\infty} y n^r(y)\, dy \right] \frac{q}{\bar{q}_s} \tag{231}$$

$$= \frac{1}{N} \frac{cQ^2}{2} + \frac{c\bar{Q}^2}{N} \left[1 + \frac{1}{N} \int_{y=0}^{+\infty} y n^r(y)\, dy \right] \left(\frac{q}{\bar{q}_s} - \frac{1}{N} \sum_s \frac{Q_s}{\bar{q}_s} \right). \tag{232}$$

Observing that $N_s(q) = N_s^r\left(\frac{q-\bar{q}_s}{\bar{q}_s}\right)$ implies $n_s(q)\,dq = \frac{1}{\bar{q}_s} n_s^r\left(\frac{q-\bar{q}_s}{\bar{q}_s}\right) dq = n_s^r(y)\,dy$. Hence,

$$x^{SUF}(q,s) = \frac{1}{N}\frac{cQ^2}{2} + \frac{c\bar{Q}^2}{N}\left[1 + \frac{1}{N}\sum_s \int_{q=\bar{q}_s}^{+\infty} \frac{q-\bar{q}_s}{\bar{q}_s} n_s(q)\,dq\right]\left(\frac{q}{\bar{q}_s} - \frac{1}{N}\sum_s \frac{Q_s}{\bar{q}_s}\right) \tag{233}$$

$$= \frac{1}{N}\frac{cQ^2}{2} + \frac{c\bar{Q}^2}{N}\left[1 + \frac{1}{N}\sum_s \left(\frac{Q_s}{\bar{q}_s} - N_s\right)\right]\left(\frac{q}{\bar{q}_s} - \frac{1}{N}\sum_s \frac{Q_s}{\bar{q}_s}\right) \tag{234}$$

$$= \frac{1}{N}\frac{cQ^2}{2} + \frac{c\bar{Q}^2}{N}\left[\frac{1}{N}\sum_s \frac{Q_s}{\bar{q}_s}\right]\left(\frac{q}{\bar{q}_s} - \frac{1}{N}\sum_s \frac{Q_s}{\bar{q}_s}\right). \tag{235}$$

Moreover, the distributional assumption that $N_s^r(r)/N_s = \alpha(\rho)$ for all s implies that:

$$Q_s = \int_{\bar{q}_s}^{+\infty} q n_s(q)\,dq \tag{236}$$

$$= \int_0^{+\infty} \bar{q}_s(1+y)\,n_s(y)\,dy \tag{237}$$

$$= \bar{q}_s \int_0^{+\infty} (1+y)\,\alpha'(y)\,N_s dy \tag{238}$$

$$= \bar{Q}_s \int_0^{+\infty} (1+y)\,\alpha'(y)\,dy \tag{239}$$

This says that $Q_s/\bar{Q}_s = \int_0^{+\infty}(1+y)\,\alpha'(y)\,dy$ is independent of s. Hence, for all s,

$$Q_s/\bar{Q}_s = Q/\bar{Q}. \tag{240}$$

Finally,

$$x^{SUF}(q,s) = \frac{1}{N}\frac{cQ^2}{2} + \frac{c\bar{Q}^2}{N}\left[\frac{1}{N}\frac{Q}{\bar{Q}}\sum_s \frac{\bar{Q}_s}{\bar{q}_s}\right]\left(\frac{q}{\bar{q}_s} - \frac{1}{N}\frac{Q}{\bar{Q}}\sum_s \frac{\bar{Q}_s}{\bar{q}_s}\right) \tag{241}$$

$$= \frac{1}{N}\frac{cQ^2}{2} + \frac{c\bar{Q}^2}{N}\left[\frac{Q}{\bar{Q}}\frac{1}{N}\sum_s N_s\right]\left(\frac{q}{\bar{q}_s} - \frac{Q}{\bar{Q}}\frac{1}{N}\sum_s N_s\right) \tag{242}$$

$$= \frac{1}{N}\frac{cQ^2}{2} + \frac{c\bar{Q}^2}{N}\left[\frac{Q}{\bar{Q}}\right]\left(\frac{q}{\bar{q}_s} - \frac{Q}{\bar{Q}}\right) \tag{243}$$

$$= \frac{1}{N}\frac{cQ^2}{2} + cQ\frac{\bar{Q}}{N}\left(\frac{q}{\bar{q}_s} - \frac{Q}{\bar{Q}}\right) \tag{244}$$

$$= \frac{1}{N}\frac{cQ^2}{2} + cQ\frac{\bar{Q}}{N}\left(\frac{q-\bar{q}_s}{\bar{q}_s} - \frac{Q-\bar{Q}}{\bar{Q}}\right). \tag{245}$$

C.3 Affine costs

SUF with absolute responsibility

As we obtained in Section C.2 Recall that under the absolute responsibility view,

$$x^{SUF}(\rho) = \frac{C(\bar{Q})}{N} + \int_{z=0}^{\rho} C'\left(\hat{Q}(z)\right) dz, \tag{246}$$

where

$$\hat{Q}(\rho) = \bar{Q} + \int_0^{+\infty} \min\{z,\rho\} n^r(z)\, dz. \tag{247}$$

Consider affine costs: $C(Q) = F + cQ$, with $F, c \in \mathbb{R}_+$. We simply have $C' \equiv c$ and, therefore:

$$x^{SUF}(\rho) = \frac{C(\bar{Q})}{N} + c\rho$$
$$= \frac{F + c\bar{Q}}{N} + c(q - \bar{q}_s).$$

SUF with relative responsibility

As before, we posit that responsibility is evenly spread across types, so that its distribution is independent of needs, $\bar{q}_s$:

$$N_s^r(\rho) = \alpha(\rho) N_s \qquad \forall s \in S, \tag{248}$$

for some increasing function $\alpha : \mathbb{R}_+ \to [0,1]$ which we take to be differentiable. As we obtained in Section C.2, recall that under the relative responsibility view,

$$x^{SUF}(\rho) = \frac{C(\bar{Q})}{N} + \frac{\bar{Q}}{N} \int_{z=0}^{\rho} C'\left(\hat{Q}(z)\right) dz, \tag{249}$$

where

$$\hat{Q}(\rho) = \bar{Q}\left[1 + \int_0^{+\infty} \inf\{z, \rho\} \alpha'(z)\, dz\right]. \tag{250}$$

Consider affine costs: $C(Q) = F + cQ$, with $F, c \in \mathbb{R}_+$. We simply have $C' \equiv c$ and, therefore:

$$\begin{aligned} x^{SUF}(\rho) &= \frac{C(\bar{Q})}{N} + \frac{c\bar{Q}}{N}\rho \\ &= \frac{F + c\bar{Q}}{N} + \frac{c\bar{Q}}{N}\frac{q - \bar{q}_s}{\bar{q}_s} \\ &= \frac{F}{N} + c\frac{1}{\bar{q}_s / \left(\bar{Q}/N\right)} q \end{aligned}$$