On the Social Organization of the Commons - An Analytical Framework

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Abstract

We study a game where agents form part of a complex social network and have to decide to adhere or not to a social norm in relation with a social dilemma. Based on moral ties, we develop an analytical approach that allows to signposts the minimal and maximal influence different types of social networks have on the agents' decision profile. We find that equilibria are locally stable within the area of network influence and their stability is augmented when informal enforcement entails costs. We determine conditions that inform policy-makers about the availability of legal and informal policy options and their effectiveness for promoting cooperativeness. The design of optimal legal and informal enforcement policies shows that less-substantial interventions are required if social networks are considered. We also show that albeit a very high share of norm-complying agent cooperativeness is unlikely to prevail if social dilemmas are perceived as not severe.

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I. Introduction

Why do we observe some communities cooperating to solve social dilemmas like producing public goods or extracting common property resources and others not? Attributes of a community or of a public good that favor cooperation have been analyzed intensively in political science, psychology, sociology, and economics (Wenegrat et al. 1996, Ostrom 2000b, Ostrom et al. 2002, Nowak 2006, Hanaki et al. 2007, West et al. 2007). Sustained cooperation is traditionally explained by compliance with social norms (Fehr et al. 2002, Fehr and Gächter 2002, Fehr and Gintis 2007). The decision to comply with social norms is voluntary if governments or communities have no power to enforce them or if it is too costly to rely on their legal or formal establishment. However, agents that adhere to social norms may apply social pressure on agents who do not adhere (Fehr et al. 2002, Ali and Miller 2016).

Understanding how social norms emerge and prevail represents an important challenge in economics. Frequently, the literature considers social norms as exogenous and does not clarify why certain behavior becomes predominant and turns into a social norm (Elster 1989, Bicchieri 1990). As a possible explanation social scientists and economists suggest that the formation of social norms is the endogenous product of social interactions (Ullmann-Margalit 1977, Axelrod 1986, Ostrom 2000a, Bicchieri and Muldoon 2014). The economic literature (Bramoullé and Kranton 2007, Jackson 2014, Acemoglu and Jackson 2017) identified the structure of social interactions (often referred as a social network) as a driving factor for the emergence and perseverance of social norms.

The literature on direct reciprocity for instance in form of social pressure suggests that repeated interaction can alleviate the social dilemma but previous work has assumed that the public good or common property resource is constant over time (Grujic et al. 2012, García and van Veelen 2016, Hilbe et al. 2018a). However, more realistic seems the case that the severity of the social dilemma as a result of the state of the public good or common property ameliorates with more cooperation and deteriorates with less cooperation. Thus, sustained cooperation allows realizing cooperative gains in the long-run that cannot be realized otherwise whereas non-cooperation leads over time to a lower stream of payoffs. Taking account of the severity of the social dilemma is underlined by a recent study by Hilbe et al. (2018b). They found that the interaction between reciprocity and payoffs related to the severity of the social dilemma is crucial: neither repeated interactions in a constant environment nor single interactions in a changing environment yield similar cooperation rates.

In this paper, we offer a qualitative and quantitative description of the combination of the characteristics of social networks, the severity of the social dilemma and the behavior of the agents

that favor cooperativeness. We present an analytical framework for studying the evolution and prevalence of a social norm based on the dynamics of two interdependent systems: a social system (a community of resource-extracting-agents presented by a social network) and a natural system (a renewable common property resource). For a wide range of different social networks the analytical framework allows studying the scope of cooperation, the design of policies and offers answers to the following two questions:

1) Economists widely agree that social interactions are important for market outcomes and for cooperation in particular. However, the question: "To what extent the presence of social interactions and its underlying pattern (type of social network) influence equilibrium outcomes and cooperativeness" has received very little attention. The economic literature has introduced the concept of a tipping point which refers in our case to the minimum share of norm-complying agents needed to induce cooperativeness among all agents (de Zeeuw and Li 2016, Nyborg et al. 2016). However, tipping points, based on the share of norm-complying agents alone, are usually an imprecise predictor for cooperativeness (Chaudhuri 2011, Tavoni et al. 2012). By taking account of the agents' rationality, the characteristics of the social network and the agents' perception of the severity of the social dilemma we do not a obtain tipping point but rather a set of frontier lines. Based on a qualitative analysis, we determine the size of this set and signpost the maximal and minimal influence different types of social networks exercise on the use of common property resources and cooperativeness. We show that locally stable second-best solutions where norm-complying and non-norm-complying agents coincides only exists within the area of network-influence. The stability of second-best solutions is augmented when social pressure is costly. The determination of the area of network influence open the door for network-orientated policies designed to increase cooperativeness by informal enforcement. These policies may comprise the foundation of a norm-complying user association, support of norm-complying agents by training workshops or other forms of privileges granted. The aim of this policy is to enhance cohesiveness among norm-complying agents and thereby increasing the peer pressure on non-norm-complying agents.

2) In this paper, we also attempt to offer an answer to the following question: "Under what conditions legal or informal enforcement policies are available to policy makers, are effective and how are they designed optimally? The paper determines initial conditions where only (i) legal enforcement, (ii) legal and informal enforcement and (iii) no enforcement policy at all are efficient policy options. Moreover, the analysis shows how the different policy options have to be designed

in order to achieve full cooperation. The results show that legal enforcement requires less-substantial interventions when social networks are taking into account. For example, the framework allows determining the number of non-compliers that need to be fined compared to the case where social networks are not considered. In the case of informal enforcement, the study allows determining the required increase in cohesiveness to attain a stable equilibrium with full cooperation. Yet, as shown in this paper, one has to keep in mind that a high number of norm-complying agents does not guarantee that an equilibrium where all agents are norm-complying is achieved it the severity of the social dilemma or scarcity of the resource is considered as not severe.

The paper is organized as follows. Section II explains the rationale of the modeling approach. In Section III we introduce the different elements of the economic framework, and in Section IV the concept for an analytical approach. In Section V we analyze the qualitative characteristics of the bioeconomic system. In Section VI we compare different enforcement policies to support equilibria and in section VII we analyze the effect of small changes in the policy instruments on the sustainable level of cooperativeness. Finally, in Section VI we draw some conclusions.

II. The rationale of the modeling approach

Within the framework of repeated non-cooperative games economists studied the behavior of agents within social networks (Haag and Lagunoff 2006, Jackson 2016). The results of these studies show that when agents have the opportunity to punish free riders, cooperation can be sustained, especially in small networks, but their applicability to real-life situations is limited because equilibria are mostly found on the basis of grim trigger strategies within the set-ups of infinitely repeated prisoner dilemma type games (Jackson 2016). Experimental works (Watabe 1992), field studies (Ostrom 1994) and theoretical models (Sethi and Somanathan 1996) found that successful cooperation is not based on grim trigger strategies because agents are leery of adopting such risky strategy in their community (Kollock 1998).

In contrast to the approach of repeated non-cooperative games, the framework of evolutionary games (Smith and Price 1973) assumes well-mixed communities where every agent is connected with all other agents (complete network). Moreover, it recognizes that agents frequently do not act fully rational. Bounded rationality seems reasonable if the social network is either large or topologically complex. Our study takes account of the complexity, vicinity and segregation patterns that occur between agents when interacting in a real-world social networks (Jackson et al. 2017). Determining the optimal strategic response to, for instance, thousands of other agents (each of whom

occupies a unique position in the social network), could stretch the assumption of rationality beyond its limits due to the complexity of the strategic decision problem.

There is a wealth of literature on social networks (Karlan et al. 2009, Jackson et al. 2012, Tavoni et al. 2012, Currarini et al. 2015) that studies the ability of a community to promote cooperativeness through reputation, trust or social pressure. In the case of social pressure, an agent that does not comply with the social norm (referred to as defector) might be exposed to social pressure from their neighbors who are complying with the norm (referred to as compliers). However, the share of compliers alone is a poor indicator for social pressure as it does not consider to what extent compliers can coordinate their action in order to augment social pressure on defectors. Thus, in the case of cooperativeness and the presence of social pressure, not only is the agents' behavior important (Tabellini 2008, Tavoni et al. 2012) but also the characteristics of the social network (Jackson 2016).

The economic literature has identified characteristics of social networks such as (i) total number of social interactions or "network density" (Karlan et al. 2009, Currarini et al. 2015), (ii) total number of agents or "network size" (Nagendra 2007, Wolitzky 2013, Currarini et al. 2015) or (iii) "the tightness of the knit" (Tabellini 2008, Enke 2019) as driving factors for cooperation. Karlan et al. (2009) refer to the network density as being responsible for strengthening the effectiveness of trust to generate and support cooperativeness. However, while network density and size are well defined on their own, there is no single metric for any social network that considers these often-diametrical characteristics. The remarkable studies by Tabellini (2008) and Enke (2019) underscore the importance of tightly-knit networks for achieving adherence to social norms. Their studies are based on either the attributes of the agents (tightness of kinship, (Enke 2019)) or the geographical distance between agents (Tabellini 2008). While these elements give rise to the notion of tightly-knit network, they do not help to define its topological structure. Most importantly, these three characteristics of social networks are mostly independent of the agents' behavior. In these studies, agents are only influenced by their neighbors' attributes or by the distance that separates them from their neighbors, however, their neighbor's choices (like adhering to the social norm or not) do not influence the agent's behavior.

In view of these observations, we employ the concept of moral ties proposed by Marco and Goetz (2020). It is based on the agents' behavior and local cohesiveness (Coleman 1988, Jackson 2016)

of a social network.[1] The agents' behavior is reflected by the share of the compliers of agent i's neighborhood, and local cohesiveness measures to what extent the neighbors of agent i are connected among each other (*friends of my friends are also my friends*). The concept of moral ties make use of a refinement of local cohesiveness as it measures to what extent the neighbors of agent i that are compliers are connected among each other, i.e., it only considers the neighbors of agent i that are compliers and not all neighbors. In this way it defines the maximal capability of compliers to interact with each other and to take collective actions if agent i is a defector (Coleman 1988). Trust and homophily (*birds of a feather flock together*) favor the decision to take collective action (Coleman 1988, Currarini et al. 2010, Bramoullé et al. 2014), but moral ties are a prerequisite (Coleman 1988) and as such it is important for the propagation and prevalence of a cooperation. For example, imagine a group of three agents (a triad) in which two agents are compliers and the third is a defector. If the three agents are connected (closed triad), one can expect (Coleman 1988) that compliers will exercise greater social pressure than if every complier were connected to a defector, but a link between compliers does not exist (open triad). In this case, compliers cannot coordinate their action because they are not connected.

The level of the remaining stock of the common property resource affects the agents' payoff. The remaining stock s is limited above by $s_{\max} \in \mathbb{R}_{>0}$ (abundance), and $s = 0$ indicates depletion. As the number of defectors within the community increases the extraction of the resource is intensified. The remaining stock not only indicates the amount of the resource left but is also a synonym for the severity of the state of the social dilemma. Thus, both; the remaining level of the natural resource and the strength of moral ties affect the intensity of the social pressure exercised (Folke et al. 2005, Ostrom 2008, Armitage et al. 2009).

Based on individual choices to adhere to a social norm or not, we present the basis for the economic model in the following section. It offers an analytical framework that combines elements of evolutionary economics and social networks that accounts for the dynamics of the social dilemma and the complexity of the interaction between agents. All of these elements are fundamental in determining the scope of cooperative behavior.

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[1] An alternative measure of cohesiveness (called "support") is employed by Jackson et al. (2012). "Support" measures whether links (social interactions) have friends or not in common. Although support and cohesiveness seem to be similar as both measures involve triads, support can be several times higher than cohesiveness. In this paper, we use cohesiveness (instead of support) because it is a well-established measure in the literature of networks and easily available for computation.

III. The economic model

We present an analytical framework for studying the evolution and prevalence of a social norm based on the dynamics and stability of two interdependent systems: a social system (a community of resource extracting agents, represented by a social network) and a natural system (a renewable common property resource), thereafter referred to as a bioeconomic system. Agents are identical and have access to the resource. They can choose between being a complier or a defector. To overcome overexploitation, compliers (denoted by C) have reached a common understanding of the characteristics of the socially optimal extraction path and may pressure defectors to follow this extraction path that could emerge as a social norm. Social pressure maybe costly. Compliers can choose to coordinate their action provided they are connected. Coordination among compliers increases the social pressure exercised.

In the following, we present the underlying assumptions of the economic model. In section III.A. we present the key components of the economic model and in section III.B its setup and timeline.

III.A Assumptions.

The model is based on the following assumptions.

- *A1. Net benefits and utilities.* All compliers have the same net benefits from resource extraction, but their utilities may differ as sanctioning costs depend on the number of neighbors that are defectors. Likewise, all defectors obtain identical net benefits from resource extraction, but have non-identical utilities which depend on the social pressure received.

- *A2. Monitoring.* All agents are perfectly informed about their neighbors' choices. There is no time delay between detecting non-compliance and exercising social pressure (informal enforcement). Legal institutions have the possibility to detect defectors but are uninformed about the underlying social network.

- *A3. Bounded rationality.* Agents do not act strategically and their decisions to be a complier or defector at time t are based only on the comparison of the current benefits and costs.

- *A4. Coordination (coalitions on the micro-scale).* Connected compliers may coordinate their action in order to increase social pressure, but do not free-ride on each other.

Assumptions 1 is self-explaining. Assumption 2 state that agents observe other agents' choices at no costs. However, legal institutions can detect defectors only if they exercise some costly effort either in form of periodical control, by monitoring devices or by giving economic incentives to compliers to reveal their choice. In any case legal institutions have no information about the underlying social network of each agent and therefore, they are uninformed about the strength of social pressure defectors receive. Assumption 3 takes account of the fact the indirect

interdependencies of every agent's choice with the choices of all other agents as a result of the large and complex network and the high dimension of the state space as a result of its continuity leads to a highly complex decision problem. For this reason, we assume as in Gale and Kariv (2003) that individual choices are based on non-strategic behavior and they are taken by comparing payoffs. We thereby formulate an indirect evolutionary process where norms, customs, and values are internalized in preferences that shape the agents' decisions (Sethi and Somanathan 1996, Sobel 2005, Dekel et al. 2007, Tavoni et al. 2012, Besley 2020)

Non-strategic behavior of the agents as the result of the complexity of the decision problem also implies that agents are not forward looking and therefore, an intertemporal preference rate does not need to be considered. Assumption 4 considers the case where connected agents may coordinate their behavior or act as a group so that the exercised social pressure is perceived as more severe by the defector. Moreover, Assumption 4 states that compliers cooperate in order to augment social pressure but they do not free-ride on each other.[2] As suggested by laboratory experiments social pressure is the result of negative reciprocity where one agent is willing to use a costly punishment when another agent transgresses (Fehr et al. 2002, Fehr and Fischbacher 2003, Dohmen et al. 2009). Compliers are not motivated only by the defectors' transgression but also because defectors' extra benefits are gains that compliers cannot realize at a later point in time. In other words, defectors inflict economic losses on compliers and therefore, social pressure can be viewed as redemption for the privation suffered by the compliers.

III.B Utility, social networks and social pressure

1.Utility. Agents' private net benefits are determined by their resource demand strategies.

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[2] If compliers were allowed to free ride on each other one had to distinguish between compliers that contribute to social pressure and non-contributing compliers. Since the agents' choice is based on the comparison of the current benefits and costs (Assumption 3) agents either choose to be a non-contributing complier or a defector but never a contributing complier. Contributing compliers only emerge if either non-contributing agents are sanctioned or if their rationality were not bounded. In the latter case they could choose to be a contributing complier because their future stream of payoffs is higher than the one resulting from other choices. However, given the complexity of the decision problem we think that bounded rationality adapts better to the considered scenario than complete rationality. Moreover, in order focus on the influence of social networks on the emergence of social norms we do not analyze the case where contributing compliers sanction non-contributing compliers. Embedding a social dilemma problem between compliers within a more general social dilemma problem between compliers and defectors would go beyond the scope of the present study.

Let $w_i^C(s(t)) \in \mathbb{R}_{\geq 0}$ denote the amount of the resource extracted at time t if agent i is a complier and $w_i^D(s(t)) \in \mathbb{R}_{\geq 0}$ if agent i is a defector. The available amount of the resource is denoted by $s(t) \in \mathbb{R}_{\geq 0}$. The values of $w_i^C(s(t))$ are the solutions of a dynamic optimization problem where a social planner maximizes the net benefits of the community over an infinite planning horizon and $w_i^D(s(t))$ are the solutions of a myopic agent that maximizes their private net benefits over the planning horizon of one year. Thus, for any given stock $s(t)$ it holds that defectors extract more than compliers, $w_i^D(s(t)) > w_i^C(s(t))$, and their net benefits, denoted by $\pi_i^D(w_i^D(s(t))) \in \mathbb{R}_{\geq 0}$, are never less than those of the compliers denoted by $\pi_i^C(w_i^C(s(t))) \in \mathbb{R}_{\geq 0}$, i.e., $\pi_i^D(w_i^D(s(t))) \geq \pi_i^C(w_i^C(s(t)))$. Since the ordinal ranking of $w_i^D(s(t))$ and $w_i^C(s(t))$ is sufficient for the proposed solution concept, the mathematical details of the optimization problems are not presented. Although compliers cannot force defectors to comply with the social norm, they can exercise social pressure at cost $\gamma_i(\cdot) \in \mathbb{R}_{\geq 0}$ (see section III.A.3 for further details). Social pressure reduces the defectors' net benefits, which in turn favors the propagation and prevalence of the social norm. We denote the social pressure agent i receives from their neighbors by the function $\omega_i(\cdot) \in \mathbb{R}_{\geq 0}$ (see section III.A.3 for further details). Given these definitions, the utility of agent i adhering to the social norm at time t, $U_i^C(t) \in \mathbb{R}_{\geq 0}$, is given by

$$U_i^C(t) = \pi_i^C(w_i^C(s(t))) - \gamma_i(\cdot). \tag{1}$$

It indicates that the utility of a complier is equal to the difference between their private net benefits[3] and the costs of social pressure.

The defectors' extra benefits represent an abstraction of the common property resource and are considered a driving force for compliers to sanction defector i. The utility of the same agent i not adhering to the social norm at time t, $U_i^D(t) \in \mathbb{R}_{\geq 0}$, is given by

$$U_i^D(t) = \pi_i^D(w_i^D(s(t))) - \omega_i(\cdot) - \theta_i, \tag{2}$$

where $\theta_i \in \mathbb{R}_{\leq 0}$ denotes a per capita subsidy, and $\theta_i \in \mathbb{R}_{\geq 0}$ a per capita fine as a result of implemented policies. Since legal institutions are not perfectly informed about the agents' choices they cannot differentiate subsidies or fines

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[3] Although the specification of the utility function implies that agents are risk neutral, we opted for this formulation to simplify the model and to concentrate on the interaction between agents in a social network.

2. Social networks. Since the position of each agent within the social network affects the agents' utility, we need to model the structure of social interactions. For this purpose, we define a social network $g = (A, L)$. It consists of a set $A = \{1, ..., n\}$, $n > 2$ of agents and a set L of links that are the unordered pairs of elements from A. The elements in the set L consist of the values of the indicatrix link function $l : A \times A \to \{0, 1\}$. For any pair of agents, (i, j), the expression $l_{ij} = 1$ indicates that the two agents are neighbors, such that $(i, j) \in L$; and $l_{ij} = 0$ indicates the two agents are not connected.[4] If $l_{ij} = 1, \forall i, j$ the network is complete, i.e., every agent is connected to all other agents. The set of neighbors of agent i is denoted by $A_i(g) = \{j : (i, j) \in L\}$ and its size $k_i = |A_i(g)| = \sum_j l_{ij}$ indicates the number of neighbors ("degree") agent i has, with $k_i \in [0, n)$. Thus, we can define local cohesiveness of the neighborhood of agent i as

$$\tau_i = \frac{\left| \{l_{uv} : u, v \in A_i(g), l_{uv} \in L\} \right|}{(k_i(k_i - 1))/2} \in [0, 1], \forall u, v \in A, u \neq v \tag{3}$$

The denominator of eq. (3) indicates the maximal number of possible links among agent i's neighbors, and the numerator indicates the number of existing links among them. The average local cohesiveness of the entire social network, denoted by τ, is quantified as $\tau = \frac{1}{n} \sum_{i=1}^{n} \tau_i$, with $\tau \in [0, 1]$. Let $A_i^C(g)$ denote the subset of compliers within agent i's neighborhood, such that $A_i^C(g) \subseteq A_i(g)$. As argued by Marco and Goetz (2020) an important characteristic of social networks to favor the exercise of social pressure and to foster cooperation is its strength of moral ties. It is based on two elements: (a) the share of compliers $c_i = \frac{|A_i^C(g)|}{|A_i(g)|} \in [0, 1]$, and (b) how close the set of compliers of the neighborhood of agent i is to being a complete network (neighborhood). Let τ_{c_i} denote the strength of local cohesiveness of compliers that can be quantified as

$$\tau_{c_i} = \frac{\left| \{l_{uv} : u, v \in A_i^C(g), l_{uv} \in L\} \right|}{(k_i(k_i - 1))/2} \in [0, 1], \forall u, v \in A, u \neq v. \tag{4}$$

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[4] We assume (i) the social network is undirected, $l_{ij} = 1 \Leftrightarrow l_{ji} = 1$; (ii) agents are not linked to themselves (no self-loops), $l_{ii} = 0$; and (iii) there is no more than one link between the pair (i, j) (uniqueness of the link).

Equation (4) represents the strength of local cohesiveness of compliers as it measures how close the subset of compliers, $A_i^C(g)$, is to being a complete network. The term τ_{c_i} signals the maximal size of the coalition compliers can form to exercise social pressure at the micro-scale, i.e., at the neighborhood level.

To illustrate the influence different network-types have on the strength of moral ties, social pressure and, consequently, on cooperativeness, we focus on three different but representative types of social networks: random, scale free and complete networks. If the formation of new links is governed by a fixed probability and independent from individual preferences the formed network is classified as a random network. If individual preferences govern the formation of links the resulting network is a scale-free network and if every agent is connected with all other agents of the network the formed network is complete.

3. Social pressure. The two key elements of social pressure agent i (defector) receives from their neighbors are given by: (i) the strength of moral ties, and (ii) the compliers' perception of the severity of the state of the social dilemma. Social pressure is denoted by the function $\omega_i\left(c_i, \tau_{c_i}^{eff}, s\right) \in \mathbb{R}_{\geq 0}$ and is based on moral ties given by the share of compliers and the effective strength of local cohesiveness of compliers, and also by the level of the remaining stock.

(i) Moral ties. As described in theoretical and empirical studies (Fehr and Gächter 2002, Calvó - Armengol and Jackson 2010, Bowles and Gintis 2011, Gächter et al. 2017), the higher the share of compliers in $A_i(g)$, the greater the social pressure exerted on agent i. We stipulate that ω_i is an increasing function in c_i, such that $\dfrac{\partial \omega_i}{\partial c_i} = \omega_{c_i}(\cdot) > 0$.[5]

A high cohesiveness of all agents in $A_i^C(g)$ does not necessarily imply that they are taking collective action and the strength of social pressure utilization may be lower than τ_{c_i}. For this reason, we consider effective local cohesiveness of compliers $\tau_{c_i}^{eff}$, e.g., the strength of collective action actually put into effect by the compliers. For the effective local cohesiveness of compliers $\tau_{c_i}^{eff}$ it holds that $\tau_{c_i}^{\min} = 0 \leq \tau_{c_i}^{eff} \leq \tau_{c_i} \leq \tau_{c_i}^{\max} = 1$. Thus, the higher the effective local cohesiveness of compliers is, the

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[5] Throughout the paper we indicate a partial derivative by a subscript of the corresponding variable.

greater the social pressure that can be exerted on defectors (Coleman 1988). We stipulate that social pressure is an increasing function in $\tau_{c_i}^{eff}$, such that $\omega_{i\tau_{c_i}^{eff}} = \dfrac{\partial \omega_i}{\partial \tau_{c_i}^{eff}} > 0$.

(ii) Remaining level of the stock. In absence of critical uncertainty regarding resource dynamics (Holling 1978, Walters 1986), we consider the stock to be a driving force for cooperation (Sethi and Somanathan 1996) and we postulate that the scarcer the resource is $(s(t) \to 0)$, the greater the social pressure the compliers exert on defector i. In other words, we stipulate that social pressure ω_i is a decreasing function in s, such that $\omega_{i_s} = \dfrac{\partial \omega_i}{\partial s} < 0$.

To summarize, the social pressure agent i (defector) receives from their neighbors, $\omega_i\left(c_i, \tau_{c_i}^{eff}, s\right) \in \mathbb{R}_{\geq 0}$, is defined by $\omega_i : \mathbb{R}_{\geq 0} \to \mathbb{R}$ a continuous, twice differentiable and increasing function in moral ties and the scarcity of the stock. Moreover, and following Bicchieri and Muldoon (2014), social pressure is (i) lower-bounded when there are no compliers at all, $\omega_i(0,0,s) = 0$, and (ii) upper-bounded, $0 \leq \omega_i(\cdot) \leq \Upsilon << \infty$, where the upper limit is equal or greater than one, i.e., $1 \leq \omega_i(1,1,0) = \Upsilon$.

After having defined the social pressure function, we specify the complier's i cost function of social pressure, $\gamma_i\left(k_i, c_i\right) \in \mathbb{R}_{\geq 0}$. It is defined by $\gamma_i : \mathbb{R}_{\geq 0} \to \mathbb{R}$ a continuous, twice differentiable and non-decreasing function in degree (k_i) and non-increasing in the share of compliers (c_i), with $\gamma_i(0,c_i) = \gamma_i(k_i,1) = 0$ and $\gamma_i(n-1,0) = \max\{\gamma_i(k_i,c_i)\}$.[6] At micro-scale, the cost function increases with the number of neighbors that are defectors.

IV. Analytical approach

The proposed game on a nonregular social network is highly complex since every agent has a particular situation in the network that influences their choice and varies with the choices of the other agents. This complexity and interdependency foreclose any qualitative analysis to describe the evolution of the game and the characteristics of its equilibria. For this reason, we propose to reduce the dimensionality of the game by approximating $c_i(t)$ and $\tau_{c_i}(t)$ for any type and size of social

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[6] We concentrate on the first-order dilemma of public goods (provision of cooperation) by assuming that the costs of exerting social pressure are strictly individual. In doing so, we avoid the second-order dilemma of public goods since compliers do not have to agree on how to divide the costs of exerting social pressure among themselves.

network by their expected values $E\left[c_i(t)\right]$ and $E\left[\tau_{c_i}(t)\right]$ that are denoted for simplicity $c(t)$ and $\tau_c(t)$.

IV.A Approximation

The values of $c(t)$ and $\tau_c(t)$ not only provide information about the average share of compliers but also allow reduce the dimensionality further. Marco and Goetz (2020) demonstrated in Theorem 1 that $\tau_c(t)$ can be approximated by $\tau c(t)^2$. It allows determining the average strength of moral ties by the characteristic of the network τ and the average share of compliers c. This approximation is very helpful since τ is a characteristic of the network that is fixed throughout the game and not like τ_c that changes as the share of compliers changes. Moreover, the dimensionality of the bioeconomic system at time t has been reduced from $(n \times n)+1$ to (1×1), given by $c(t)$ and $s(t)$.

The proposed approximation paves the way for a qualitative analysis but the question remains whether the approximation is robust? As a response to this question one can evaluate the approximation error by determining the difference between the approximated social pressure function, denoted by $\hat{\omega}\left(c(t), \tau c^2(t), s(t)\right)$, and the approximated cost of sanctioning, denoted by $\hat{\gamma}\left(k, c(t)\right)$, with the average social pressure $\dfrac{1}{n}\sum_{i=1}^{n}\omega\left(c_i(t), \tau_{c_i}^{eff}(t), s(t)\right)$ and average sanctioning costs $\dfrac{1}{n}\sum_{i=1}^{n}\gamma_i\left(k_i, c_i(t)\right)$ respectively. Obviously, if $\tau_{c_i}^{eff} < \tau_{c_i}$ one obtains an additional approximation error since we approximate $\tau_{c_i}(t)$ and not $\tau_{c_i}^{eff}(t)$ by $\tau_c(t)$. Notwithstanding, this approximation error is not important for a qualitative analysis since the approximation of $\tau_c(t) = \tau c^2(t)$ leaves us with the parameter τ that is constant throughout the game. Hence, a variation of this parameter can reflect either a change in the structure of the underlying social network, or a reduction in the willingness to take collective actions to sanction defectors. In the latter case the value of the parameter has to be chosen below the cohesiveness of the social network, since $\tau_{c_i}^{eff}(t) < \tau_{c_i}(t) \approx \tau c^2(t)$.

Proposition 1: [Robustness of the approximation]

The error of the approximation of the social pressure function $\omega(\cdot)$ and of cost of sanctioning $\gamma(\cdot)$ by $\hat{\omega}\left(c(t), \tau c^2(t), s(t)\right)$ and $\hat{\gamma}\left(k, c(t)\right)$ respectively vanishes as the underlying social network

tends (i) to be complete, (ii) to be dense or regular, or (iii) to have a homogenous distribution of compliers and defectors.

Proof: See Appendix A.I

The conditions formulated in Proposition 1 guarantee that the social pressure each defector i receives and the sanction costs each compliers bear are similar to the average social pressure and the average sanction costs respectively. However, even when these conditions are not met the approximation error can be negligible small provided that the distribution of social pressure and sanctioning costs at the micro-level is symmetric around theirs means so that they cancel out at the macro-level.

At macro-scale it holds that the sanctioning cost are equal to zero when the average agent has no link, or if either all agents are defectors or compliers, i.e., $\hat{\gamma}(0,c) = \hat{\gamma}(k,0) = \hat{\gamma}(k,1) = 0$. Thus, $\hat{\gamma} : \mathbb{R}_{\geq 0} \to \mathbb{R}$ is a continuous, twice differentiable and non-decreasing function in average degree (k) but for small/large average values of share of compliers (c) it approaches asymptotically zero and has a maximum at intermediate values, $c_\gamma \in (0,1)$, such that $\hat{\gamma}(n-1, c_\gamma) = \max\{\hat{\gamma}(k,c)\}$. The latter case corresponds to the situation where the maximal number of compliers and defectors are neighbors. By Assumption 2 legal institutions are uninformed about the agent's social network so that they cannot adjust a fine according to the social pressure a defector receives or according to the costs of social pressure a complier has. Hence, fines or subsidies are uniformly applied such that $\forall i \in A, \; \theta_i = \hat{\theta}$.

A qualitative analysis of the bioeconomic system has important implications in policy design as it contributes to answering the question as to what extent social networks influence the equilibrium outcomes and how they affect the availability and efficiency of policy options (see sections VI and VII).

Based on the assumption of bounded rationality (A3) agents choose in our dynamic game the action that offers a higher utility than the average utility obtained by the other agents of the social network. Thus, the evolution of cooperativeness is given by

$$\frac{\partial c}{\partial t} \equiv \dot{c} = c\left(U^C - U\right) = c\left(U^C - \left(cU^C + (1-c)U^D\right)\right) = c(1-c)\left(U^C - U^D\right), \tag{5}$$

where U denotes the average utility of all agents, i.e., of U^C and U^D.[7] Writing out the definitions of U^C and U^D and collecting term shows that

$$\dot{c} = c(1-c)\left(\left(\pi^C - \pi^D\right) + \hat{\omega} + \hat{\theta} - \hat{\gamma}\right). \tag{6}$$

In order to facilitate the analysis and interpretation of equation (6) we convert defector´s extra benefit as reference point with a value of one. Thus, dividing equation (6) by defector´s extra benefit yields

$$\frac{dc}{dt} = \dot{c} = \left(\hat{\omega}(c(t), \tau c(t)^2, s(t)) - 1 + \hat{\theta} - \hat{\gamma}(k, c(t))\right)c(t)(1 - c(t)). \tag{7}$$

Since the interpretation of the functions and parameters $\hat{\omega}, \hat{\theta}$ and $\hat{\gamma}$ remains unchanged we maintain their notation in order to reduce the notational burden of the study. Equation (7) shows that the share of compliers increases if the social pressure is on average higher than the sum of the defectors' extra benefits, normalized to one, legal enforcement in form a fine/subsidy and the costs of social pressure, i.e., $\hat{\bar{\omega}} \geq 1 - \hat{\theta} + \hat{\gamma}$. This inequality is in line with the assumption of bounded rationality as it implies that social pressure in the absence of legal enforcement is greater than the cost of social pressure. The term $c(1-c)$ relates to the interaction rate between compliers and defectors. It graduates the evolution of cooperativeness, $\dot{c}$, and is minimal if the share of compliers is either close to zero or one.

Cooperativeness relates also to the evolution of the stock that is described by

$$\frac{\partial s}{\partial t} \equiv \dot{s} = R\left(s(t)\right) - \sum_{i=1}^{cn} w_i^C\left(s(t)\right) - \sum_{j=cn+1}^{n} w_j^D\left(s(t)\right) = R(s(t)) - cnw^C(s(t)) - (1-c)nw^D(s(t)), \tag{8}$$

where $R\left(s(t)\right) \in \mathbb{R}_{\geq 0}$ denotes the stock-dependent growth rate of the renewable resource. This variable could present for instance a forest or fishery under a common property regime or an aquifer where the recharge is constant so that $R\left(s(t)\right) \equiv R$.

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[7] Note that equation (5) is similar to the two-strategy replicator as in Bowles and Ginis (1998). However, the replicator dynamic assumes that the social network is always complete and the evolution of a stock variable is not considered. The two-strategy replicator assumes that all agents, although constantly connected, meet with an identical and constant probability, which in turn conditions the propagation of the agents' behavior within the community. In equation (5) such an assumption is not necessary as our model is influenced by the structure of the agents' links and the severity of the social dilemma.

IV.B Setup and timeline of the game on social networks

The game is described as follows:

1. Setup. The game is initiated as follows:

- The structure of the social network $g \in (A, L)$ is static throughout the run of the game but its initial structure can be modified between different runs of the game. Its structure is characterized by the average local cohesiveness, $\tau \in [0,1]$, which is fixed during the game.

- The size of the network is finite but large and constant during the run of the game, i.e., $|A| = n < \infty$.

- The game begins at time $t = 0$ with an initial share of compliers, $c(t = 0) \equiv c_0 \in (0,1)$, and the available stock, $s(t = 0) \equiv s_0 \in (0, s_{max}]$.

2. Timeline. The timeline of the model within each period is as follows.

- At each moment of time $t \in [1, T]$, the agent i in g will face a binary choice, $\{C, D\}$, where C represents compliance and D non-compliance with the social norm. As a result, the average share of compliers $c(t) \in (0,1)$ evolves over time.

- At time t the agents choose the action that offers a higher utility than the average utility of their neighbors, given the share of compliers in their neighborhood, and the available stock. All agents choose simultaneously and take the other agents' choices as given.

- The time horizon of the game T is finite but very long. It ends when $t = T$ or $s = 0$ (depletion).

IV.C. Evolution of cooperativeness and social norms

Given the structure of the social network $\tau \in [0,1]$, and any set of initial conditions, $c(0) = c_0 \in (0,1)$ and $s(0) = s_0 \in (0, s_{max}]$, cooperativeness as a result of a social norm has emerged over time if $c(t = T) \in (0,1]$; with $t \in [0,T]$. On the contrary, if $c(t = T) = 0$ and/or $s(t = T) = 0$, cooperativeness has not emerged.

First- and second-best solutions. Our interest is in identifying conditions for sustaining the all-complier equilibrium that by definition of $w_i^C (s(t))$ is the first-best solution as it defines the socially

optimally outcome. Alternatively, if the first-best solution cannot be achieved, we are interested in defining conditions that allow sustaining an interior equilibrium where compliers and defectors coexist (second-best solution).

The existence of interior equilibria of the bioeconomic system leads to the definition of a "frontier line" in Section V. The slope and the set of possible locations of the frontier line allow us to signpost the minimal and maximal influence social networks have on the use of common property resources and cooperativeness. This information is highly relevant for policy design as it provides policy-makers with information about the availability and effectiveness of legal and informal enforcement policies – see Sections VI and VII.

Definition 1 [Equilibrium conditions]: *The stationary values for the social system and the natural system are given by $c*$ and $s*$ respectively. An overall equilibrium of the non-linear bioeconomic system is obtained when $c*$ and $s*$ exist, and it holds that $\dot{s} = \dot{c} = 0$.*

For any fixed social network, the dynamics of the non-linear bioeconomic system is governed exclusively by $\dot{c}$ and $\dot{s}$. An overall equilibrium is characterized by $R(s*) = cnw^C(s*) + (1-c)nw^D(s*)$ and $c = 0$, $c = 1$ or $\hat{\omega}(c, \tau^{eff}c^2, s) = 1 - \hat{\theta} + \hat{\gamma}(k, c)$. The values $c* = \underline{c} = 0$ (lower), $c* = \tilde{c} \in (0,1)$ (interior), $c* = \overline{c} = 1$ (upper) comply with the equation $\dot{c} = 0$, and $s*$ with the equation $\dot{s} = 0$. The all-complier equilibrium is achieved when $c* = \overline{c} = 1$, and the all-defector equilibrium is achieved when $c* = \underline{c} = 0$.

V. Qualitative analysis of the bioeconomic system

Equations (7) and (8) allow us to determine (1) the influence social networks have on equilibrium outcomes and cooperativeness, and to identify (2) the dynamics of the frontier line and the basins of attraction of the bioeconomic system. Furthermore, it allows us to determine (3) the influence the stock variable has on the agents' behavior.

V.A. Social network structure

Analytically, the approximation $\hat{\omega}\left(c, \tau c^2, s\right)$ extends existing approaches[8] for studying the influence different types of networks have on the qualitative behavior of the bioeconomic system. To guide the intuition behind our analysis, we focus on representative types of social networks[9]: random networks, scale free networks and complete networks. In random networks, links are formed with a fixed and usually small probability so that the agents' capacity of interaction is conditioned by this probability. As a result, all agents are similarly poorly connected. In scale-free networks, some agents tend to be more (but not completely) connected than the others and therefore they interact more often. These networks offer a better match to real-world social networks, especially when communities are large. In complete networks, every agent interacts with everyone else. Given the agent's limited capacity to be permanently linked to a large number of people, complete networks are likely to be a good match for small but not for large communities.

Marco and Goetz (2020) demonstrate in the proof of Proposition 4 that the range of the average local cohesiveness in random networks (RN) is given by $\tau_{RN} \in \left[\dfrac{2}{n-1}, \dfrac{2}{\sqrt{n}}\right]$, so that it tends to zero for n sufficiently large, in different versions of scale-free networks (SF) it is given by $\tau_{SF} \in \left[0, \dfrac{4}{5}\right]$, and in complete networks (CN) average local cohesiveness is maximal, i.e., $\tau_{CN} = 1$. The effects social networks have on the equilibrium outcomes and cooperativeness can be seen from equations (7) and (8). The growth rate of the share of compliers increases as c and/or τ^{eff}, bounded from above by τ, increases which in turn affects the strength of moral ties and the level of the stock. Hence, social pressure is lowest in a random network and highest in a complete network for a given stock of the resource and a share of compliers and its willingness to collaborate.

The wide variety of real-world network topologies (scale-free networks) opens the door to design network-orientated policies that aim to enhance social pressure by increasing average local cohesiveness τ (see Sections VI and VII).

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[8] To date to our best knowledge studies that analyzed the influence of networks on the qualitative behavior of economic models have focused on random networks or simple geometric figures like a line, star, triad or circle, in the absence of a stock variable.

[9] For more details about the formation and evolution of social networks see Jackson and Rogers (2007).

V.B. Frontier and border line

While the boundary (lower and upper) stationary values of the $\dot{c}=0$ isocline, stated in Definition 1, are straightforward, the intermediate case, $c^* = \tilde{c} \in (0,1)$, is more intriguing as it allows for a stationary value where compliers and defectors coexist. The isocline $\hat{\omega}(c, \tau^{eff} c^2, s) - 1 + \hat{\theta} - \hat{\gamma}(k, c) = 0$ presents an iso-social pressure line where agents are indifferent between the compliance or non-compliance. In this respect, it presents a frontier line for the agents' choice. The following definition determines all possible locations of the frontiers lines in the s, c - plane.

Definition 2 [Frontier line]:

The set $\mathcal{F} = \bigcup_{\tau^{eff} \in [0,1]} \mathcal{F}_{\tau^{eff}} \left\{ s, c \in \mathbb{R}^2 \, \middle| \, \hat{\omega}(c, \tau^{eff} c^2, s) - 1 + \hat{\theta} - \hat{\gamma}(k, c) = 0 \right\}$ defines the area with all possible frontier lines in the s, c - plane. The elements of the set $\mathcal{F}$ are the different sets $\mathcal{F}_{\tau^{eff}}$.

The frontier lines with the minimal and maximal τ^{eff} are given by $\mathcal{F}_{\tau^{eff}=0} \equiv \hat{\omega}(c, 0, s) - 1 + \hat{\theta} - \hat{\gamma}(k, c) = 0$ and $\mathcal{F}_{\tau^{eff}=1} \equiv \hat{\omega}(c, c^2, s) - 1 + \hat{\theta} - \hat{\gamma}(k, c) = 0$ respectively. A frontier line moves to the north-west in the s, c - plane with an increase in τ^{eff}, so that $\mathcal{F}_{\tau^{eff}=1}$ is to the left of $\mathcal{F}_{\tau^{eff}=0}$ - see Figure 1 below for an illustration. The frontier line of a random network constitutes the right-hand-side boundary of the set $\mathcal{F}$, whereas the frontier line of a complete network the left-hand-side boundary if compliers collaborate. In this way, the sets $\mathcal{F}_{\tau^{eff}=1}$ and $\mathcal{F}_{\tau^{eff}=0}$ signpost the maximal and minimal influence the type of social network has on the cooperativeness. The frontier line presents a generalization of the concept of a tipping point (threshold) in the share of compliers. The concept of a tipping point emerges as a special case if (i) the interior equilibrium is unstable, (ii) the social network is complete and (iii) there is either no stock or the level of the stock is constant over time. The generalization of the concept of a tipping point and offers an answer to the first question raised in the introduction shows that the share of compliers as a single aspect of social interaction is an imprecise predictor for cooperation. An argument that is also supported by the evidence found by experimental economists (Chaudhuri 2011). If only the level of the stock were constant over time we would obtain a completely horizontal frontier line because the stock does not influence social pressure. This horizontal line shifts positionally to the left or to the right as a result of an increase or decrease in cohesiveness respectively.

The isocline $\dot{s}=0$ offers the borderline between resource depletion (overexploitation) and resource replenishment (abundance). The slope of the borderline and frontier line is determined by

Proposition 2 below. The intersection of the frontier line and the borderline yields the overall equilibrium of the bioeconomic system (c^*, s^*) in the s, c - plane.

Proposition 2: [Slope of the isoclines]

The slope of the borderline and the frontier line (nullclines) in the s, c - plane are given by

$$\left.\frac{\partial s}{\partial c}\right|_{\dot{s}=0} = \frac{-\left[n\left(w^D(s)-w^C(s)\right)\right]}{\frac{\partial R(s)}{\partial s}-cn\frac{\partial w^C(s)}{\partial s}-(1-c)n\frac{\partial w^D(s)}{\partial s}} = \begin{cases} >0, \text{ if } \frac{\partial R(s)}{\partial s}-cn\frac{\partial w^C(s)}{\partial s}-(1-c)n\frac{\partial w^D(s)}{\partial s}<0 \\ <0, \text{ if } \frac{\partial R(s)}{\partial s}-cn\frac{\partial w^C(s)}{\partial s}-(1-c)n\frac{\partial w^D(s)}{\partial s}>0 \end{cases} \tag{9}$$

$$\left.\frac{\partial s}{\partial c}\right|_{\dot{c}=0} = \frac{-\left[\hat{\omega}_c+\hat{\omega}_{\tau_c}2\tau c-\hat{\gamma}_c\right]}{\hat{\omega}_s} \gtreqless 0, \tag{10}$$

Proof: Applying the implicit function theorem to the equations (7) and (8) respectively, yields the results of Proposition 2.[10] □

Proposition 2 shows that the borderline (eq. 10) is upward sloping in the equilibrium if changes in resource growth are less than the increase in extractions as a result of an increase in the stock. This would be the case if the growth rate of the resource is independent from the stock, e.g., the natural recharge rate of an aquifer. Similarly, the borderline is downward sloping if resource is very responsive to changes in the stock but extraction is not, for example if extraction costs are completely independent from the remaining level of the stock. The frontier line is upward sloping if the social pressure function is more responsive to an increase in the share of compliers than the cost of social punishment. This is the always the case if the social punishment costs are either fixed, or if the costs of social punishment are hardly responsive to an increase in the share of compliers. If these conditions are not met the frontier line can be upward or downward sloping in the s, c - plane.

For the sake of concreteness, we concentrate on the case where the borderline line is upward sloping and the frontier line is either upward or downward sloping. Other combinations of the slopes of the nullclines can also be analyzed within the presented analytical framework. However, as they do not offer any new qualitative insights we do not present them in this study

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[10] As defined previously with respect to ω_i we also obtain for $\hat{\omega}$ that,

$$\hat{\omega}_c = \frac{\partial\hat{\omega}}{\partial c} > 0, \hat{\omega}_{\tau_c} = \frac{\partial\hat{\omega}}{\partial\tau_c} > 0, \hat{\omega}_s = \frac{\partial\hat{\omega}}{\partial s} < 0.$$

V.C. Basins of attraction and policy options

Consistent with the first- and second-best solutions defined above, we want to identify the policies and conditions that best favor cooperation among the agents. This objective goes back to the second question posed in the introduction. Sections V.C aims to answer this question. Concretely, we want to define the conditions under which no-enforcement or informal enforcement, in form of network-orientated policies, or legal enforcement, in form of economic-orientated policies (e.g., fines or subsidies), may be best applied. We start by defining the basins of attraction.

Definition 3 [Basins of attraction]: *For a given social network, e.g., for any given value of*

$$\tau^{eff} \in [0,1], \quad the \quad sets \quad \mathcal{R} = \bigcup_{\tau^{eff} \in [0,1]} \mathcal{R}_{\tau^{eff}} = \left\{ s,c \in \mathbb{R}^2 \,\middle|\, \hat{\omega}(c,\tau c^2,s) - 1 + \hat{\theta} - \hat{\gamma}(k,c) < 0 \right\} \quad and$$

$$\mathcal{G} = \bigcup_{\tau^{eff} \in [0,1]} \mathcal{G}_{\tau^{eff}} = \left\{ s,c \in \mathbb{R}^2 \,\middle|\, \hat{\omega}(c,\tau c^2,s) - 1 + \hat{\theta} - \hat{\gamma}(k,c) > 0 \right\} \; define \; \mathcal{R} \; as \; the \; basin \; of \; attraction \; for$$

the all-defector equilibrium ($c^* = \underline{c} = 0$), and $\mathcal{G}$ as the basin of attraction for the all-complier equilibrium $\left(c^* = \overline{c} = 1 \right)$.

1. No-enforcement corridor – It is independent of the type of network. Values of c, τ and s located in set $\mathcal{G}_{\tau^{eff}=0}$ indicate that the social norm has emerged as a result of strong moral ties together with the threat of a deteriorating state of the natural resource. For values located in $\mathcal{G}_{\tau^{eff}=0}$, the stable equilibrium outcome is independent of any type of social network. The social norm and social pressure are sufficient for achieving the all-complier equilibrium. We refer to this set as the no-enforcement corridor. Policy-makers have to be aware that a high share of compliers alone may not be sufficient for the abandonment of enforcement policies – see Proposition 4 below.

2. Informal-enforcement corridor - It depends on the type of network. The set $\mathcal{F}$ defines the lower and upper limits of network-orientated policies as it depends of the type of social network. The objective of informal enforcement (network-orientated policies) is to strengthen indirect moral ties, τ_c, by increasing τ^{eff}, and, thus, these policies are able to reduce the minimum share of compliers necessary for cooperativeness. For this reason, we refer to set $\mathcal{F}$ as the network-policy or informal-enforcement corridor. In the context of a natural resource, one can imagine, with respect to network-orientated policies, a compliers' association and/or a non-governmental support organization being founded, or seminars and workshops being organized (Cumming 2018). Likewise, community leaders (referred to hubs in network terminology) may help to create links between her neighbors to increase the cohesiveness in the social network. The regulator or the

compliers themselves may take the initiative for these policies. Moreover, within set $\mathcal{F}$, network-orientated policies may be complemented or replaced by legal enforcement policies.

3. Legal-enforcement corridor – It is independent of social interactions at all. Values of c, τ and s located in set $\mathcal{R}_{\tau^{eff}=1}$ indicate the non-emergence of the social norm and are independent of the type of network. The equilibrium outcome for initial values located in $\mathcal{R}_{\tau^{eff}=1}$ corresponds to the all-defector equilibrium, and thus, it is identical to the situation where agents do not interact at all. Moral ties have not been sufficiently formed within the community and therefore no-enforcement or informal enforcement policies have to be discarded. Policy-makers are left only with the option of legal enforcement, for instance, direct regulations, if possible, or economic incentives. For this reason, we refer to set $\mathcal{R}_{\tau^{eff}=1}$ as the legal-enforcement corridor. The objective of legal enforcement is to strengthen direct moral ties by increasing c.

If agents were not interacting and no legal enforcement were exercised the equilibria would be given by $c^* = \underline{c} = 0$, and by $c^* = \overline{c} = 1$. In this case the basin of attraction of the all defector equilibrium were given by the set $\left\{ s, c \in \mathbb{R}^2 \mid (-1)c(1-c) < 0 \right\}$, i.e. the entire admissible s,c plane except $c = 1$. In other word the traditional result of the tragedy of the commons.

The three corridors identify the policy options available and are analyzed in Section VI.

Proposition 3: [Social networks and location of equilibria]

The size of the set $\mathcal{F}$ not only signposts the maximal and minimal influence of the types of social networks on cooperativeness but also the possible area of location of the second-best solution (stable equilibrium where compliers and defectors coexist).

Proof: Following Definition 1, an equilibrium exists if and only if $\dot{c} = 0$. According to Definition 2 the set $\mathcal{F}$ contains all the cases where interior stationary values of the share of compliers exists, i.e., $c^* = \tilde{c} \in (0,1)$. Hence, any equilibrium of the bioeconomic system, $\dot{s} = \dot{c} = 0$, where compliers and defectors coexist has to be part of the set $\mathcal{F}$. □

In other words, Proposition 3 indicates that any second-best solution, if it exists, is always located in the area of network influence where $(s^*, c^* = \tilde{c}) \in \mathcal{F}$ holds.
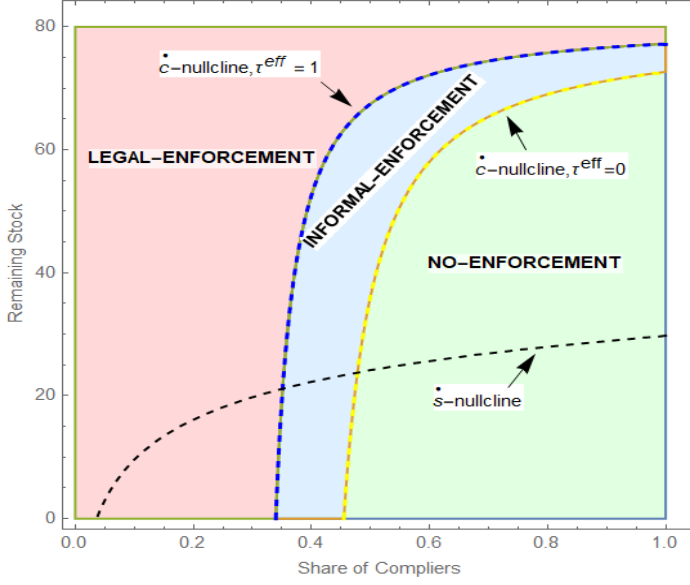
Figure 1 Basins of attraction of equilibrium points and the availability of enforcement policy options

Figure 1 illustrates the location of the two nullclines of $\dot{c}, \dot{s}$ for the case where legal enforcement is absent and social pressure is costless, i.e., $\hat{\gamma}(k,c) = 0$. For example, it could present the case of a water user association where farmers have their own well and extract water from a common property aquifer for irrigation. The employed functions for Figure 1 are obtained from the solution of a mathematical optimization model programed in GAMS and a subsequent econometric analysis of the output of the GAMS model. The required agronomic, economic and hydrological input data for the model was obtained from published data about the Western La Mancha aquifer in Spain. The online material presents detailed information about chosen values for the parameters and the specification of the employed function $\hat{\omega}$, w^C and w^D. Moreover, given the normalization of the defector's extra benefit to one we have limited social pressure by twice this value, i.e. $\omega_i(\cdot) \leq \Upsilon = 2$. Other values for Υ are possible but its choice does not change the qualitative results of our analysis.

In Figure 1 the area $\mathcal{R}$ is marked in red and the area $\mathcal{G}$ in green. The area between the frontier line $\mathcal{F}_{\tau^{eff}=1}$ and $\mathcal{F}_{\tau^{eff}=0}$ signposts the maximal and minimal influence the type of social network has on

possible coexisting equilibria. It is marked in light blue and its size depends on τ^{eff} , the effective strength of cohesiveness of all agents.[11]

VI Implications of the analytical framework for policy design

In this section we analyze first the evolution and stability of the bioeconomic system as reference points for the design of enforcement polices and second determine the design of optimal polices for stable cooperation.

VI.A Stability analysis

The intersections of the nullclines of $\dot{c},\dot{s}$ presents an interior equilibrium where compliers and defectors coexist. In Appendix III we present a stability analysis for possible interior equilibria of the bioeconomic system. It shows that an equilibrium-point where the slope of the frontier line at this point is superior to the positive slope of the border line is a saddle point (Observation A2). A numerical analysis confirms this result since the determinant of the Jacobian matrix of the equations (7) and (8) evaluated at the equilibrium point $\tilde{c}_2 = (c,s) = (0.40916, 22.43928)$ is negative with a value of -0.08968. However, since the analysis in Appendix III is based on the linearization of the nullclines evaluated at an interior equilibrium point it presents only an approximation. It describes the dynamics of the bioeconomic system in the neighborhood of the equilibrium point and little can be deduced about the dynamics of the bioeconomic system beyond the neighborhood of the equilibrium point. For this purpose, we present in Figure 2 the phase diagram for the entire c,s space. It illustrates that the stable branch of the saddle point corresponds to a movement along the $\dot{c}$ nullcline and the unstable branch any point off the $\dot{c}$ nullcline. Given the narrow margin of the stable path, the chances that arbitrarily chosen initial values of c,s , even if they are initially on the stable branch, belong to a path that ends up at the interior equilibrium is very limited. Figure 2 also illustrates that the equilibria, $c^* = 0, c^* = 1$ are stable. Hence, the bioeconomic system is characterized by the three stable equilibria, $c^* = 0, c^* = 1, c^* = \tilde{c}$. Each of them is indicated by a black circle.

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[11] The strength of cohesiveness of all agents τ depends on the availability of links between agents whereas the strength of cohesiveness of compliers τ_c depends on the availability of links between agents and the share of compliers. Both elements of τ_c influences in the strength of social pressure.
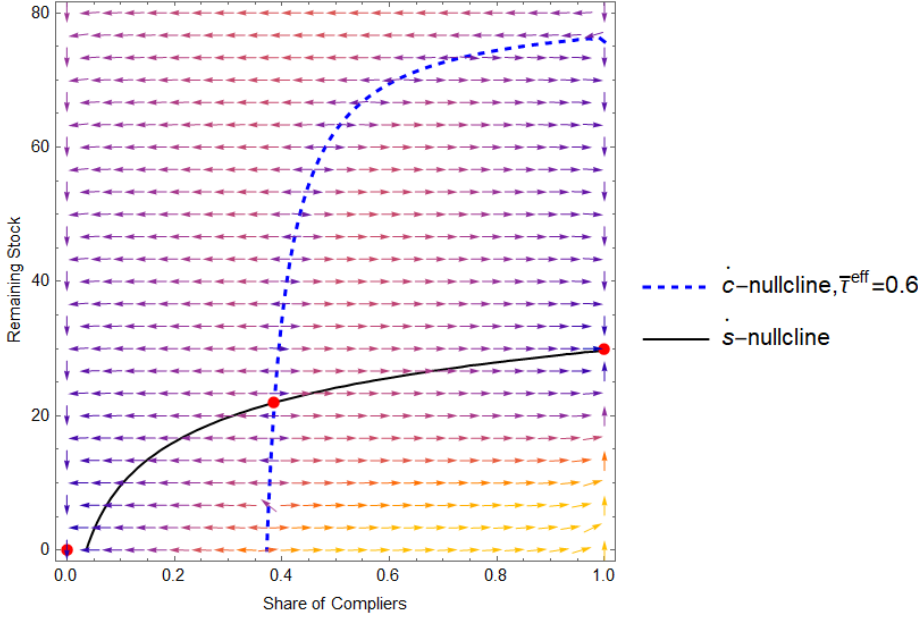
Figure 2. Phase diagram of the bioeconomic system, nullclines and equilibrium points in the absence of policy interventions and any costs of social pressure.

Next, we analyze the influence of the average costs of social pressure on the equilibrium outcome. Let us assume that they are given by $\hat{\gamma}(k,c) = \upsilon(k)c(1-c)$, where $\upsilon(k)$ is strictly positive and constant throughout the game. Based on the specification of the function $\hat{\gamma}(\cdot)$ the costs of social pressure are bell-shaped and maximal if $c = 0.5$. Note that the sum of the defector's extra benefits and average costs of social pressure exceeds the maximal value of social pressure $\Psi = 2$, if $c = 0.5$ and $\upsilon > 4$.

Figure 3 depicts the location of the two nullclines of $\dot{c}, \dot{s}$ and the phase diagram of the bioeconomic system for the case where social pressure is given $\hat{\gamma}(k,c) = 4.3c(1-c)$ and legal enforcement is absent, i.e., $\hat{\theta} = 0$. It shows that the bioeconomic system is characterized by the two boundary equilibria $c^* = 0, c^* = 1$ and three interior equilibria, denoted by $\tilde{c}_1, \tilde{c}_2$ and $\tilde{c}_3$. The phase diagram of the bioeconomic system shows that the equilibria $c^* = 0, c^* = 1$ are stable. Appendix III demonstrates that an interior equilibrium point is a sink if the slope of the frontier line at this point is negative while the slope of the border line is positive (Observation A1). Thus, equilibrium point $\tilde{c}_2$ is a sink and the equilibrium points $\tilde{c}_1$, and $\tilde{c}_3$ are saddle points if the slope of the frontier and border line are upward sloping but the former one is more pronounced than the later one (Observation A2). A numerical analysis of the determinant of the Jacobian matrix of the equations

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(7) and (8) evaluated at the equilibrium points $\tilde{c}_1 = (0.17, 15.01), \tilde{c}_2 = (0.36, 21.53)$ and $\tilde{c}_3 = (0.63, 26.04)$ confirm the analytical results since the determinants are given respectively by -0.00838, 0.00248, -0.00293 and the trace of the Jacobian matrix evaluated at the equilibrium point $\tilde{c}_2$ is -0.25637. Furthermore, Figure 3 also demonstrates that the basin of attraction of the interior equilibria $\tilde{c}_1, \tilde{c}_3$ is very small since its narrow margin does not show up in the phase diagram as its grid size is larger than the basin of attraction. Thus, the chances that the bioeconomic system starting at any arbitrarily chosen values ends up at the interior equilibria $\tilde{c}_1, \tilde{c}_3$ are very limited. However, Figure 3 shows that the basin of attraction of the equilibrium $\tilde{c}_2$ is quite large and the chances that an arbitrarily chosen point c, s forms the starting point of a path that leads to the equilibrium $\tilde{c}_2$ or to all complier equilibrium are significant. A sensitive analysis of the parameter values indicates that a stable interior equilibrium with a significant large basin of attraction emerges only if informal enforcement is available and the maximal average costs of social pressure exceed social pressure for certain values of c.
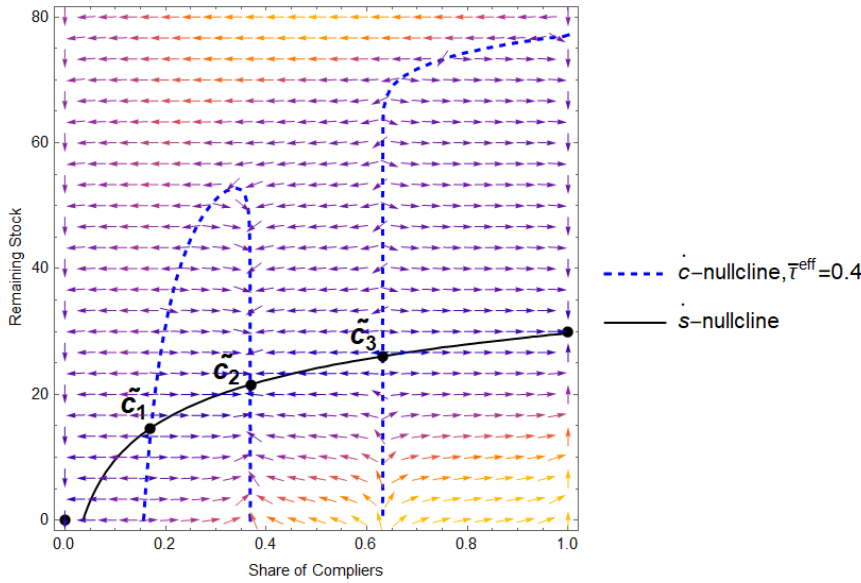

Figure 3. Phase diagram of the bioeconomic system, nullclines and equilibrium points where the costs of social pressure are strictly positive but policy interventions are absent. [12]

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[12] The $\dot{c}$-nullcline does not exist for $c < 0.16$ because social pressure is very close to zero even if $s = 0$. It also does not exist for $0.42 < c < 0.57$ because the bell-shaped costs of social pressure exceed social pressure even if it is at its maximal value of Υ. Thus, there does not exist a value of s that satisfies the equation $\dot{c} = 0$ given that $0.42 < c < 0.57$.

VI.B. Design of optimal enforcement policies for cooperation

Let us assume that the initial values of s and c are given by s_t and c_t, and the cohesiveness of the underlying social network is given by $\overline{\tau}^{eff}$. Policy options in form of legal and informal enforcement are available if s_t and c_t are elements of the set $\mathcal{F}_{\tau^{eff} < \overline{\tau}^{eff} \in [0,1]}$ that is marked in light blue in Figure 1. This set consists of all frontier lines where $\tau^{eff} < \overline{\tau}^{eff}$ and consequently the point c_t, s_t is to the left of the frontier line with $\tau^{eff} = \overline{\tau}^{eff}$. Therefore, the first-best solution cannot be achieved. Instead the system tends to the all-defector equilibrium.

Legal enforcement: Policy makers have two options. They can either aim to change increase the number of compliers or they can restrict the access to the resource (e.g. in fishery seasonal or temporal restriction are frequently applied).

(i) Change in the number of compliers: For an arbitrarily small ε, policy makers can employ legal enforcement so that the share of compliers increases from c_t to $c_{LE} + \varepsilon$, where $c_{LE} \in \mathcal{F}_{\tau^{eff} = \overline{\tau}^{eff}}$, i.e., the frontier line that level of cohesiveness is $\overline{\tau}^{eff}$ and passes through the point c_{LE}, s_t. If the share of compliers is equal to $c_{LE} + \varepsilon$ Figure 4 shows that the point c_{LE}, s_t is to the right of the frontier line $\mathcal{F}_{\tau^{eff} = \overline{\tau}^{eff}}$ and the bioeconomic system tends to the all-complier equilibrium. Legal enforcement is effective if the regulator has detected at least $c_{LE} + \varepsilon - c_t$ defectors. Imposing a fine on this number of defectors opens the pathway for the first-best solution. This example clearly demonstrates that not it is not necessary to fine all defectors if social pressure is present. Only the number of defectors that are missing to reach the basin of attraction of the all-complier equilibrium need to be fined.

(ii) Restrictions on the access to the resource: This type of policy is effective if it holds that the initial point $\hat{c}_t, \hat{s}_t \in \mathcal{R}$ and there exist a value s_R for which holds that $\hat{c}_t, s_R \in \mathcal{G}$. In this case the policy maker can restrict the access to the resource to s_R. Frequently, there will exist more than one value of s_R and the policy is likely to choose the highest value of s_R in order to minimize the related loss in net benefits for the community. In Figure 4 the access to the initially available resource $\hat{s}_t$ has been restricted to $\hat{s}_R$. The restricted access is illustrated by the dashed green line.
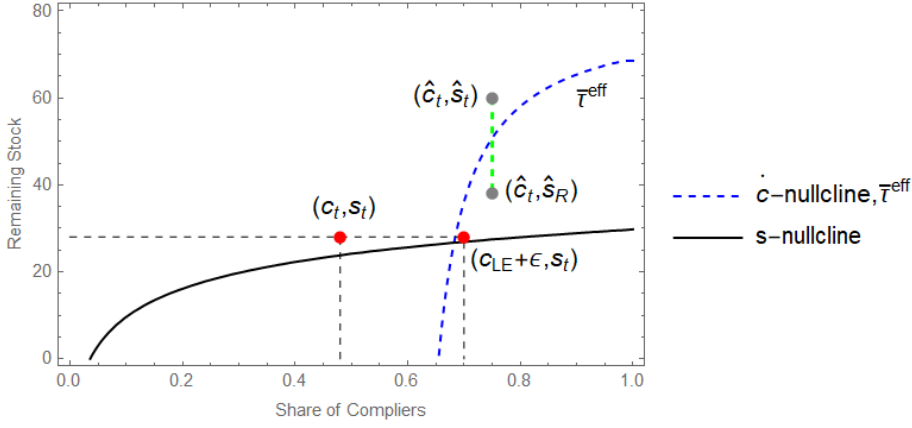
Figure 4. Legal enforcement by imposing a fine on $c_{LE+\varepsilon} - c_t$ defectors or by restricting access to the resource to $\hat{s}_R$.

Informal enforcement: Alternatively, the policy makers can resort to informal enforcement which raises cohesiveness from $\overline{\tau}^{eff}$ to $\tau_{IE}^{eff} + \varepsilon \leq 1$ with $\tau_{IE}^{eff} > \overline{\tau}^{eff}$. We denote the frontier with lines $\tau^{eff} = \overline{\tau}_{IF}^{eff}$ that passes through the point c_t, s_t, by $\mathcal{F}_{\tau^{eff} = \tau_{IE}^{eff}}$. A further increase of cohesiveness by ε yields the frontier line $\mathcal{F}_{\tau^{eff} = \tau_{IE}^{eff} + \varepsilon}$ that is located to the left of the frontier line $\mathcal{F}_{\tau^{eff} = \tau_{IE}^{eff}}$. Consequently, as shown in Figure 5, the initial point c_t, s_t, is to the right of the frontier line $\mathcal{F}_{\tau^{eff} = \tau_{IE}^{eff} + \varepsilon}$ and the bioeconomic system tends to the all-complier equilibrium.
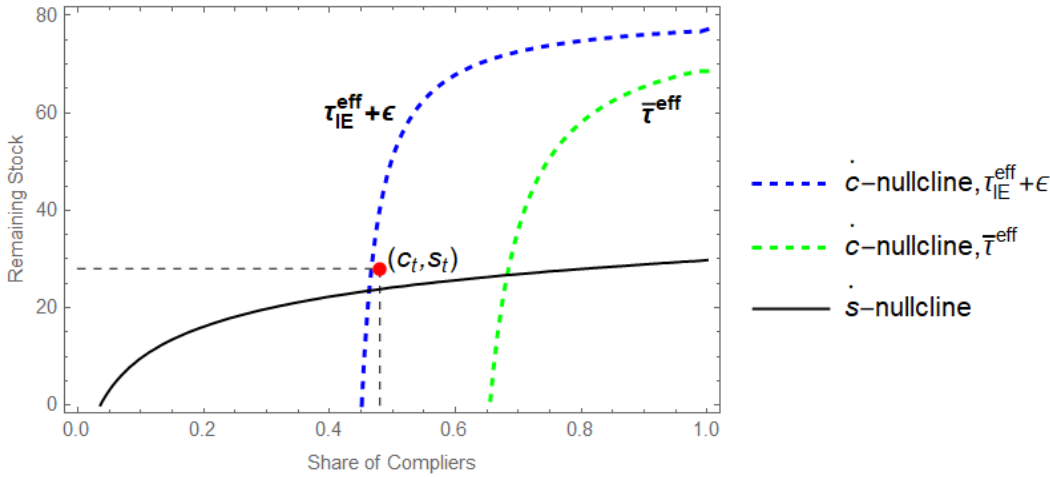

Figure 5. Informal enforcement by strengthening cohesiveness beyond τ_{IE}^{eff} .

Mix of legal and informal enforcement: Apart from legal-only or informal-only enforcement polices policymakers can also employ a mix of enforcement policies. For this purpose, policymakers can choose any value of $c \in (c_t, c_{LE})$. Let us denote the chosen value by c_{LEIE}. Hence, policymakers have to implement a legal enforcement policy that raises the share of complier slightly above c_{LFIF}, that

we denote by $c_{LEIE} + \varepsilon$. At the same time and they have to implement an informal enforcement policy that increases the cohesiveness from $\bar{\tau}^{eff}$ to $\tau^{eff}_{LEIF} < 1$. The value of τ^{eff}_{LEIF} is defined by the set $\mathcal{F}_{\tau^{eff} = \tau^{eff}_{LEIE}}$ .that passes through the point c_{LFIF}, s_t. Hence, as shown in Figure 6 the new point $c_{LFIF} + \varepsilon, s_t$ is to the right of the frontier line $\mathcal{F}_{\tau^{eff} = \tau^{eff}_{LEIE}}$ and the dynamics of the bioeconomic system lead to the all-complier equilibrium.

For any value of $c_{LFIF} \in (c_t, c_{LE})$ policymakers can determine the costs of the each of the two enforcement policies. Policymakers are likely to choose the point $c_{LFIF} \in (c_t, c_{LE})$ where the mix of enforcement polices has the lowest overall costs. The optimal least-cost mix of enforcement policies depends on the fixed and variable costs of informal enforcement policies and the fixed and variable costs of the legal enforcement policies. If detection of defectors requires a cost-intensive monitoring system legal enforcement is clearly in disadvantage compared to informal enforcement. Since the regulator cannot distinguish a priori between compliers and defectors the monitoring costs of legal enforcement have to be shared equally between all agents. Thus, it does not affect the utility difference between compliers and defectors and therefore, also not the choices taken by the agents. Hence, monitoring costs of legal enforcement would not produce any new qualitative results and for this reason they are not included in the analysis of the bioeconomic system. Yet, legal enforcement generates revenues from the collected fines and implies sanctioning costs. The influence these revenues and costs have on the design of the optimal enforcement policy is summarized in Observation 1.
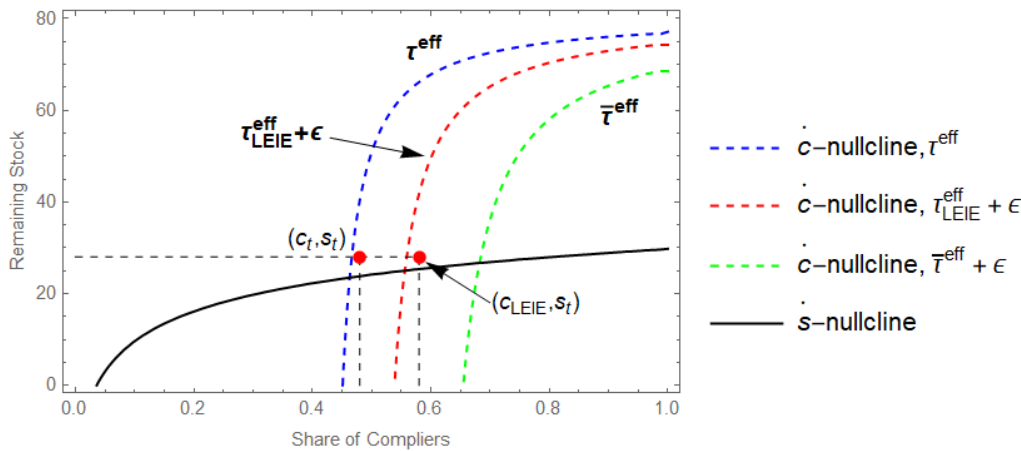

Figure 6 Implementation of a mix of legal and informal enforcement policies by imposing a fine on $c_{LEIE} - c_t$ defectors and by strengthening cohesiveness beyond τ^{eff}_{LEIE} .

Observation 1 [Policy design]: *If the initial values of s and c are elements of the set $\mathcal{F}_{\tau^{eff} < \bar{\tau}^{eff} \in [0,1]}$ legal or informal enforcement policies on their own or any combination are feasible options to achieve full cooperation. Depending on how the collected fine and the sanctioning costs of legal enforcement compare to the fix and variable costs of informal enforcement policies determines the design of the optimal enforcement policy.*

The following proposition shows the importance of the remaining stock for achieving or failing in cooperation.

Proposition 4 [High share of compliers]: *Even a very high share of compliers, but strictly less than one, may be insufficient for cooperation when the social dilemma problem is perceived as not severe.*

Proof: See Appendix V and Figures 2 and 3. □

Proposition 4 shows that abundance of the resource may lead to insufficient social pressure that cannot be compensated by any high share of compliers. In this case, even with a high share of compliers, the all-defector equilibrium cannot be avoided. The high share of compliers that does not support an all-complier equilibrium is situated in Figures 2 and 3 in the upper right corner.

VI.C. Incidence of policy instruments on the second-best equilibrium

Given that the regulator has no information about the underlying social network of each agent it is only possible for her to design enforcement policies that are identical either for all compliers or for all defectors. The two policy options informal and legal enforcement are only available if the initial values of c, τ and s form part of set $\mathcal{F}$. If these initial values form part of the set $\mathcal{R}_{\tau^{eff} = 1}$, legal enforcement is the only available policy option and if they form of the set $\mathcal{G}_{\tau^{eff} = 0}$, no policy intervention is required. Thus, we concentrate our analysis on the effect uniform policies have on the equilibrium outcome where compliers and defectors coexist. Moreover, we assume, for now, that the enforcement costs for both policies are identical.

The first policy scenario is based on informal enforcement aimed at intensifying social pressure by increasing τ. As demonstrated by Marco and Goetz (2020) the expected strength of indirect moral ties relates directly to the type of network and thus the analysis of the changes in these policy instruments shows the influence the network type has on social pressure and finally on cooperativeness.

The second policy scenario is based on legal enforcement in the form of policy instruments like a uniform per capita fine on defectors. These policy instruments can be evaluated by varying the term θ. We evaluate the effect of a change of the fine $\theta \in \mathbb{R}_{\geq 0}$ on the share of compliers and the remaining stock. The analysis of the different policy instruments is based on comparative statics of the equations.[13]

$$\dot{c} = \left(\hat{\omega}(c, \tau c^2, s) - 1 + \hat{\theta} - \hat{\gamma}(k,c) \right) = 0, \tag{11}$$

$$\dot{s} = R(s) - cnw^C(s) - (1-c)nw^D(s) = 0. \tag{12}$$

Definition 4 [Direct complier-effect and indirect complier-stock-effect]: *An increase in the share of compliers leads to a change in social pressure in two different ways. On the one hand, we observe a direct complier-effect on social pressure that is defined by $\Omega_c = \left(\hat{\omega}_c + \hat{\omega}_{\tau_c} 2\tau c - \gamma_c \right)$. On the other hand, we observe a change in the stock that affects social pressure indirectly. The indirect complier-stock-effect on social pressure is defined by $\Omega_s = \hat{\omega}_s \left. \dfrac{\partial s}{\partial c} \right|_{\dot{s}=0}$.*

The effect of legal and informal enforcement policies on the share of compliers in equilibrium where compliers and defectors coexist is stated in the following two propositions:

Proposition 5 [Network-orientated policies and equilibria]: *Network-orientated policies imply an increase in τ, which leads in the case of partial cooperation given by an equilibrium in form of a sink to an increase in the share of compliers, and in case of an equilibrium in form of a saddle to an increase (decrease) in the share of compliers if the sum of the direct complier-effect $\left(\Omega_c \right)$ and indirect complier-stock-effect $\left(\Omega_s \right)$ on social pressure is negative (positive).*

Proposition 6 [Legal policies and equilibria]: *A fine on defectors leads in the case of an equilibrium in form of a sink to an increase in the share of compliers, and in the case of an equilibrium in form of saddle to an increase (decrease) in the share of compliers if the sum of the direct complier-effect $\left(\Omega_c \right)$ and indirect complier-stock-effect $\left(\Omega_s \right)$ on social pressure is negative (positive).*

Proof of Propositions 5 and 6: See Appendix V

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[13] Since we focus on a interior equilibria the term $c(1-c)$ in equation (7) can be neglected.

Proposition 5 and 6 indicates the sensitivities of equilibria where compliers and defectors coexist as a result of a modification of informal and legal enforcement policies. While the results for an equilibrium in form of a sink are unambiguous the results for an equilibrium in form of a saddle depend on the strength of the direct complier-effect and indirect complier-stock-effect.

VIII. Conclusions

This paper offers an analytical framework for studying agents' behavior that extract a common property resource and interact on a social network. Based on the combination of the agents' strategy choices and the topological characteristics of social networks the paper employs the concept of moral ties.

In the previous literature the share of compliers has been identified as an important driver for cooperation and is what gave rise to the concept of a tipping point. The paper introduces a new concept that we refer to as the frontier line. It presents a generalization of the idea of a tipping point which emerges as a special case if the network is complete and there is either no stock or the level of the stock is completely constant over time. Most importantly, the frontier line signposts the maximal and minimal influence different types of social networks can exercise on the equilibrium outcome. It also indicates that the type of social network does not play any role once cohesiveness is maximal and any type of network forms part of the basin of attraction for the all-complier equilibrium.

Based on an approximation approach for the share of compliers and cohesiveness, the paper determines the qualitative properties of the possible equilibria of the bioeconomic system that is governed by the network topology, the agents' strategy choices and the evolution of the common property resource. The analysis shows that the all-defector and all-complier equilibrium are stable. It further defines the condition for locally stable equilibria where defectors and compliers coexist. Despite their local stability they might not be of interest for policy if their basin of attraction is very limited as shown for the case of the Western La Mancha aquifer. However, a stable equilibrium with partial cooperation and a significant larger basin of attraction in mixed actions may exists if the costs of social punishment are sufficiently high. This finding suggests that subsidizing the social pressure costs may be counterproductive.

The study offers policy-makers information about the availability, effectiveness and efficiency of different policy options to enforce norm-complying behavior for different initial values of the stock, share of compliers and cohesiveness. If the initial conditions form part of a set of frontier lines policy makers can employ any combination of legal and informal enforcement policies. The results of the analysis allow designing the optimal combination of these two policy options. It is worth

accentuating that legal and informal enforcement policies in the presence of social network are less invasive then in the absence of social networks. The analysis also indicates that even a high share of norm-complying agents may be insufficient for cooperation when the social dilemma is perceived as not severe.

With respect to future work the proposed model offers an analytical framework for an empirical evaluation of the effect of social interactions on the use of common property resources, for example the estimation of the conditional probability for the compliance of a social norm depending on the stock, share of compliers, cohesiveness, socioeconomic factors and fixed effects. Likewise, the model may serve as a guideline for the design of efficient policies to improve cooperativeness. As related to an extension of the theoretical framework one may take into account that legal and informal enforcement may crowd in or crowd out each other. For example, legal enforcement may displace informal enforcement if the agents are less committed to pressure defectors in the presence of a government policies. Likewise, it may occur that legal enforcement encourages compliers to exercise more social pressure since it reinforces their existing beliefs towards the social norm (Bowles and Polania-Reyes 2012).

Appendix

I. Proof of Proposition 1.

In any network $g = (A, L)$, the approximation of social pressure based on the mean field theory yields $\hat{\omega}(\cdot) \equiv \hat{\omega}(E[c], E[\tau_c], s) = \hat{\omega}(c, \tau c^2, s)$. However, the average social pressure in the entire social network is given by $\omega(\cdot) = \frac{1}{n} \sum_{i=1}^{n} \omega_i(c_i, \tau_{c_i}, s)$. If social pressure is non-linear, we can prove by Jensen's inequality that $\hat{\omega}(\cdot) \neq \omega(\cdot)$. Given the approximation, it implies that we have to consider the possible error $\varepsilon = \dfrac{\sum_{i=1}^{n} |\omega_i(\cdot) - \hat{\omega}(\cdot)|}{n} = |\omega(\cdot) - \hat{\omega}(\cdot)|$. In the following, we present three cases when the error of the approximation is zero or very low.

Case 1. The social network is complete. When the social network g is complete, all agents are fully connected and thus, $\forall i \in A$, $c_i = c$ and $\tau_i = \tau = 1$ always hold. The strength of direct and indirect moral ties is identical for all agents. In complete networks, all agents receive identical social

pressure and thus the mean field approximation, $\hat{\omega}(\cdot)$, equals the average social pressure of the social network, such that $\forall i \in A,\ \omega_i(\cdot) = \hat{\omega}(\cdot) = \omega(\cdot)$.

Case 2. The social network is sufficiently dense or regular. When the social network is sparse, $0 < |L| << \dfrac{n(n-1)}{2}$, where $|L|$ represents the size of the set of links, the strength of direct and indirect moral ties is not identical among agents. As the two parameters (c_i, τ_i) are heterogeneous in the social network, $\omega_i(\cdot)$ fluctuates non-symmetrically around $\hat{\omega}(\cdot)$ and thus, $\varepsilon > 0$. In this case, a numerical analysis is needed to calculate the exact value of $\omega(\cdot)$ and the exact error of the mean field approximation. However, we can easily observe that the magnitude of the error is low when the social network is dense or regular. Let us denote by $Var(c) = \dfrac{1}{n}\sum_{i=1}^{n}(c_i - c)^2$ the variance of the strength of direct moral ties and by $Var(\tau_c)$ the variance of the strength of indirect moral ties within the social network. The second variance depends on two independent variables (network cohesiveness and agents' actions) and thus:

$$Var(\tau_c) = Var(c^2)Var(\tau) + Var(c^2)\left(E[\tau]\right)^2 + Var(\tau)\left(E\left[c^2\right]\right)^2. \tag{13}$$

The denser the network, $|L| \to \dfrac{n(n-1)}{2}$, the higher the average local cohesiveness $\tau \uparrow 1$. If $\tau \uparrow 1$, not only $Var(\tau) \to 0$ but also $Var(c) \to 0$. This means that all neighborhoods have similar strengths of direct and indirect moral ties as $|L|$ increases. In this case, the error of the mean field approximation tends to zero, $\varepsilon \downarrow 0$. Similarly, when social networks are regular, they present similar values of local cohesiveness, $\tau_{\min} \sim \tau_{\max}$ (e.g., lattices or random networks, such that $Var(\tau) \to 0$, and therefore, the magnitude of the error term is also low (see equation A2).

Case 3. The distribution of behavior within the social network is homogeneous. Very low (or very high) strength of direct moral ties, $c \downarrow 0$ (or $c \uparrow 1$), indicates that behavior is homogeneous within the social network, and consequently $Var(c) \to 0$. In this case, the second variance also tends to zero, $Var(\tau_c) \to 0$, and thus, the error of the mean field approximation tends to zero, $\varepsilon \downarrow 0$ (see equation A2). □

III Stability Analysis.

We have the following dynamical system

$$\dot{c} = \hat{\omega}(c, \tau c^2, s) - 1 + \hat{\theta} - \hat{\gamma}(k,c)c(1-c) \tag{14}$$

$$\dot{s} = R(s(t)) - cnw^C(s(t)) - (1-c)nw^D(s(t)) \tag{15}$$

For the $\dot{c}$-isocline we can immediately identity the equilibria $\underline{c} = 0$ and $\overline{c} = 1$. Thus, we concentrate in the following on the case of the equilibria where compliers and defectors coexist, i.e., $\dot{c} = \hat{\omega}(c, \tau c^2, s) - 1 + \hat{\theta} - \hat{\gamma}(k,c) = 0$. The Jacobian matrix J of the system (14) and (15) is given by

$$J = \begin{pmatrix} \dfrac{\partial \dot{c}}{\partial c} & \dfrac{\partial \dot{c}}{\partial s} \\ \dfrac{\partial \dot{s}}{\partial c} & \dfrac{\partial \dot{s}}{\partial s} \end{pmatrix} = \begin{pmatrix} \hat{\omega}_c + \hat{\omega}_{\tau_c} 2\tau c - \hat{\gamma}_c & \hat{\omega}_s \\ n\left(w^D(s) - w^C(s)\right) & \dfrac{\partial R(s)}{\partial s} - cn\dfrac{\partial w^C(s)}{\partial s} - (1-c)n\dfrac{\partial w^D(s)}{\partial s} \end{pmatrix}$$

For the eigenvalues λ_1, λ_2 of the dynamical system it holds that $\lambda_1 * \lambda_2 = \det J$, and $\lambda_1 + \lambda_2 = \text{Trace } J$. Based on the previous structure of the functions $\hat{\omega}, w^C, w^D, \hat{\gamma}$ and R the evaluation of the sign of the determinant of J shows that:

$$\det J = (\pm) * (-) - (+)(-) \gtrless 0.$$

Given the mathematical structure of the employed functions a stable equilibrium in form of a sink can be identified. For this case, it is required that $\det J > 0$ and $\text{Trace } J < 0$.

The evaluation of the determinant shows that

$$\det J = \left(\hat{\omega}_c + \hat{\omega}_{\tau_c} 2\tau c - \hat{\gamma}_c\right)\left(R_s - cnw_s^C - (1-c)nw_s^D\right) - n\left(w^D(s) - w^C(s)\right)\hat{\omega}_s \quad \text{which implies for}$$

$\det J > 0$ and taking account of the negative sign of $\hat{\omega}_s$ that

$$\det J = \frac{\left(\hat{\omega}_c + \hat{\omega}_{\tau_c} 2\tau c - \hat{\gamma}_c\right)}{\hat{\omega}_s} > \frac{n\left(w^D(s) - w^C(s)\right)}{R_s - cnw_s^C - (1-c)nw_s^D}. \tag{16}$$

Multiplying this inequality by -1 yields

$$\frac{-\left(\hat{\omega}_c + \hat{\omega}_{\tau_c} 2\tau c - \gamma_c\right)}{\hat{\omega}_s} < \frac{-n\left(w^D(s) - w^C(s)\right)}{R_s - cnw_s^C - (1-c)nw_s^D}. \tag{17}$$

so that according to equations (9) and (10) the left and right side of the inequality can be related to the sign of the slope of the frontier line and border line respectively. Hence, the existence of a sink

requires that $\left.\dfrac{\partial s}{\partial c}\right|_{\dot c=0} < \left.\dfrac{\partial s}{\partial c}\right|_{\dot s=0}$ which is satisfied if $\left.\dfrac{\partial s}{\partial c}\right|_{\dot c=0} < 0$ since $\left.\dfrac{\partial s}{\partial c}\right|_{\dot s=0} > 0$. We can relate this

inequality directly to the elements of the matrix J. It implies that $\dfrac{\partial \dot c}{\partial c} < 0$. Moreover, the last

inequality guarantees that $\dfrac{\partial \dot c}{\partial c} + \dfrac{\partial \dot s}{\partial s} = \lambda_1 + \lambda_2 = \text{Trace } J < 0$. Alternatively, we can consider the case

where $\quad 0 < \left.\dfrac{\partial s}{\partial c}\right|_{\dot c=0} < \left.\dfrac{\partial s}{\partial c}\right|_{\dot s=0}$. However, in this case it cannot be guaranteed that

$\dfrac{\partial \dot c}{\partial c} + \dfrac{\partial \dot s}{\partial s} = \lambda_1 + \lambda_2 = \text{Trace } J < 0$ and it has to be imposed as an additional condition.

Observation A1: *An equilibrium point of the bioeconomic system is characterized by a sink if the frontier line at this point is downward sloping, and the border line at this point is upward sloping.*

Similarly, we can analyze the conditions for an equilibrium in form of saddle, where it is required that $\det J < 0$. We can derive the necessary condition from equation (17) by reversing the inequality sign. Thus, in order to guarantee the existence of a saddle point it has to hold that

$\dfrac{-\left(\hat\omega_c + \hat\omega_{\tau_c} 2\tau c - \gamma_c\right)}{\hat\omega_s} > \dfrac{-n\left(w^D(s) - w^C(s)\right)}{R_s - cnw_s^C - (1-c)nw_s^D}$. Thus, an equilibrium in in form of a saddle exists if

the positive slope of the $\dot c$-isocline is greater than the positive slope of the $\dot s$-isocline.

Observation A2: *An equilibrium point of the bioeconomic system is characterized by a saddle if the frontier line and border line at this point are upward sloping but the slope of the frontier line is greater than the slope of the border line.*

IV. Proof of Proposition 4.

Let $\left(\mathcal{R}, \leq\right)$ be a set whose elements are totally ordered by the relation $\leq$. Since, the definition of $\mathcal{R}$

is given by $\mathcal{R} = \bigcup_{\tau^{eff} \in [0,1]} \mathcal{R}_{\tau^{eff}} = \left\{s, c \in \mathbb{R}^2 \mid \hat\omega(c, \tau^{eff} c^2, s) - 1 + \hat\theta - \hat\gamma(k, c) < 0\right\}$ we can define for a

given $s \in [0, s_{\max}]$ a lowest upper bound (supremum) of c in $\mathcal{R}$ but not a máximum. The supremum

of c is denoted by $\left.c^{\sup}\right|_s \in [0,1]$, with $s \in [0, s_{\max}]$, and given by

$$\left.c^{\sup}\right|_{s \in [0, s_{\max}]} = \begin{cases} 1 & , & (s, c=1) \in \mathcal{R}_{\tau^{eff}=1} \\ c \in \mathcal{F}_{\tau^{eff}=1} & , \text{if } s \in \mathcal{F}_{\tau^{eff}=1} \text{ and } \left(c > c \in \mathcal{F}_{\tau^{eff}=1}\right) \notin \mathcal{R} \\ \nexists & , & \text{if } s \notin \mathcal{R}_{\tau^{eff}=1} \end{cases} \tag{18}$$

Equation (18) defines the supremum of c for the cases where the frontier line $\mathcal{F}$ is not defined for the upper part, $\left(c^{\text{sup}} = 1\right)$, the lower part, $\left(c^{\text{sup}} = 0\right)$, and when it is defined for the intermediate part, $\left(c^{\text{sup}} \in [0,1]\right)$, of the (s,c)-plane, see Figure 1 for an illustration. Within the (s,c)-plane, an cooperation may not emerge when the following three conditions hold: (i) none of the three sets $\mathcal{R}$, $\mathcal{F}$ and $\mathcal{G}$ is empty; (ii) for a given $s \in [0, s_{\text{max}}]$, the supremum $c^{\text{sup}}\big|_{s}$ exists and $\mathcal{R}$ is continuous in c, such that $\left(s, c \leq c^{\text{sup}}\big|_{s}\right) \in \mathcal{R}$; and (iii) the set $\mathcal{R}$ includes the point $c = 0$, such that $(s,0) \in \mathcal{R}$. In the case where all three conditions hold, there exists a path in the (s,c)-plane that lies entirely within the set $\mathcal{R}$ and connects a possible high but insufficient level of cooperation, $0 < c < c^{\text{sup}}\big|_{s} \leq 1$, with no cooperation at all, $c = 0$. This path represents all the cases where the state of the social dilemma is perceived as not severe by the agents. Following this path, it can be observed that, even when the bioeconomic system departs from high level of cooperation, anti-social behavior and/or depletion of the natural resource may occur. If the three conditions hold, the higher are the values of s that belong to set $\mathcal{R}$ the higher the value of $c^{\text{sup}}\big|_{s}$ that cannot prevent the emergence of the all defector equilibrium. □

V. Proof of Proposition 5 and 6.

Since the isocline of $\dot{s} = 0$ (equation (12)) depends on c and s but not on τ and θ, we can redefine the isocline and write s in form of $s(c)$. Next, within the $\dot{c} = 0$ isocline (equation (11)), we replace s by $s(c)$ so that the unknowns of equation (11) are a function of τ, θ, i.e., $s\left(c(\tau,\theta)\right)$ and $c(\tau,\theta)$. Based on the implicit function theorem, we obtain for the isocline of $\dot{c} = 0$ that

$$\frac{\partial c}{\partial \tau} = \frac{-\hat{\omega}_{\tau_c} c^2}{\Omega_c + \Omega_s} \tag{19}$$

$$\frac{\partial c}{\partial \theta} = -\frac{1}{\Omega_c + \Omega_s} \cdot \tag{20}$$

We can determine the signs of the equations (19) and (20) with the help of Table A1.

Table A1. Determination of the signs of the equations (19) and (20) based on the equations (9), (10) for the case where the borderline is upward sloping and the equations (11) and (12).

$\frac{\partial \dot{c}}{\partial c} = \Omega_c = \hat{\omega}_c + \hat{\omega}_{\tau_c} 2\tau c - \gamma_c$	$\Omega_s = \hat{\omega}_s \left. \frac{\partial s}{\partial c} \right|_{\dot{s}=0}$	$\Omega_c + \Omega_s$
Negative (sink)	negative	negative
Positive (saddle)	negative	positive or negative

Table A1 together with equations (19) and (20) show that the sign of the direct complier-effect, Ω_c, and the indirect complier-stock-effect, Ω_s, is negative for an equilibrium in form of sink while for an equilibrium in form of saddle the terms Ω_c and Ω_s have opposite signs. Thus, for the equilibrium in fom of a sink the effect of a change in informal or legal enforcement on a new equilibrium is unambiguous but for an equilibrium in form of a saddle it depends on the sum $\Omega_c + \Omega_s$. $\square$

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Online Material

Numerical analysis and specification of the functions employed

For the numerical application we need to define resource demand function for compliers and defectors. These functions serve as reference for the behavior of compliers and defectors with respect to the extraction of the resource. The compliers' resource demand function at time t is given by the solution of the social planner's optimization problem over the planning horizon from t to T. Mathematically the social planner's resource demand function at time t is given by:

$$w_i^C(s(t), t) = \underset{w_i^C(s(t))}{\arg\max} \int_t^T \sum_{i=1}^n e^{-r_s v} \pi_i^C(w_i^C(s(v)), s(v)) dv, \tag{21}$$

subject to

$$\dot{s} = R(s(v)) - \sum_i^{cn} w_i^C(s(v)) - \sum_{cn+1}^n w_j^D(s(t)), \ s(t) = s_t \tag{22}$$

where r_s denotes the social planner's inter-temporal discount rate. The defector's resource demand function at time t is given by the solution of the defector's private optimization problem that can be formulated as

$$w_i^D(s(t)) = \underset{w_i^D(s(t))}{\arg\max} \ e^{-r_p} \pi^D(w_i^D(s(t)), s(t)), \tag{23}$$

where r_p denotes the defector's inter-temporal discount rate. We consider that defectors are very shortsighted and therefore r_p is set equal to zero. We assume that the social planner's expectation at time t of the defectors' future extraction, $\bar{w}_i^D$, is based on their current level of extraction, i.e., $\bar{w}_i^D = w_i^D(s(t))$. The social planner's resource demand function is the solution to a feed-back strategy, since agents can observe the level of stock remaining. In other words, the agents modify their extraction profile based on the amount of the natural resource they observe.[14]

As an example of high policy relevance, we focus on the western La Mancha aquifer situated within the upper Guadiana basin (inland region of Castilla La Mancha, Spain) and extending over 5000km^2

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[14] In the game agents have to decide whether to become a complier or defector and both of them are not forward-looking as a result of the complexity of the social network. Yet, the complier's reference behavior for the extraction of water is based on the solution of the social planner's problem which inherently implies forward looking behavior.

(see Figure O1). The constant overdraft of this aquifer has led to a variety of policy measures designed to curb its deterioration and comply with the European Union Water Framework Directive (European Comission 2000, Blanco-Gutiérrez et al. 2011).

Figure O1: Geographical location of the western La Mancha aquifer
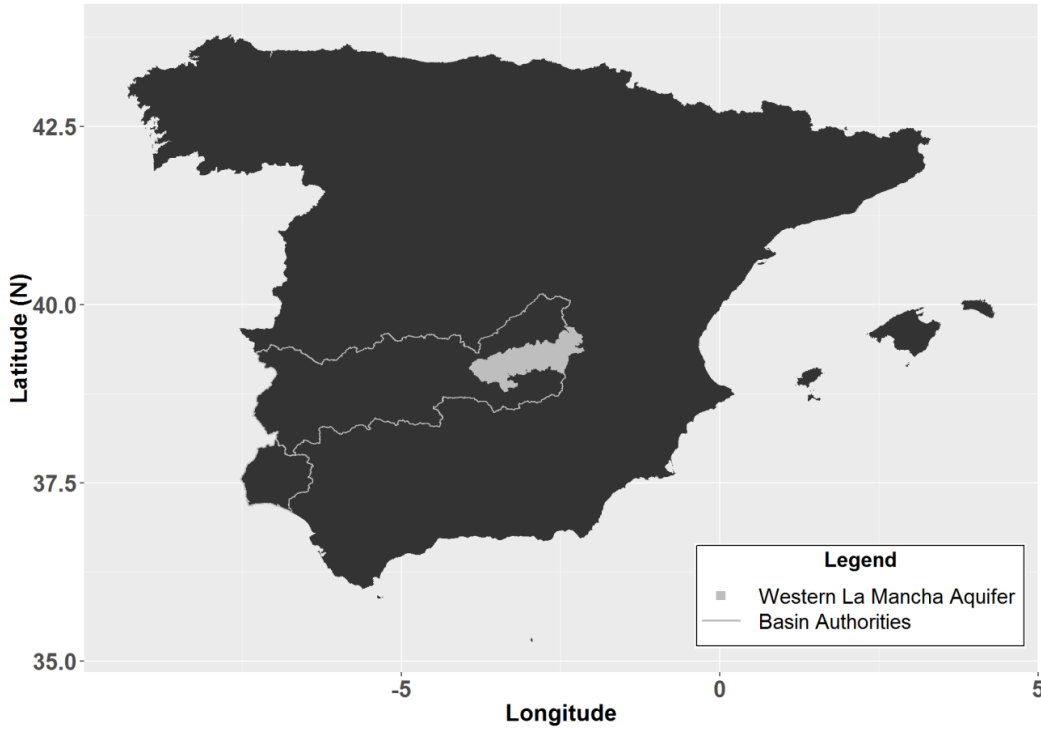
Hydrological model

La Mancha aquifer is fed naturally and we assume that the recharge is constant, i.e., the term $R(s(t))$ is equal to R. Since the extraction is measured in m^3 and the depth of the water table in m, we need to adapt equation (8) by introducing the conversion factor μ that expresses the change in the depth of the water table as a result of the water extraction. Moreover, a part of the water extracted for irrigation percolates back into the aquifer. Following Esteban and Albiac (2010, 2011), we set this return rate, denoted by ψ, equal to 20%. Thus, the dynamics of the natural resource can be written as

$$\dot{s}(t) = \mu \left(R - \psi \left(\sum_{i}^{cn} w_i^C(t) + \sum_{cn+1}^{n} w_i^D(t) \right) \right). \tag{24}$$

Esteban and Albiac (2011) report that the land cultivated for agricultural production comprised in 2007 191,400 ha. Esteban and Albiac (2010, 2011) calculated that the irrigation practices lead to a gross extraction of 1km^3 of water from the aquifer and extended the irrigated area by 62,000 ha. Consequently, the water table dropped by 32 meters over 12 years, i.e., a decrease of 2.6666 m per year. The gross extraction per m^3 of water per ha is given by 1,000,000.000m^3/253400ha= 3946.329 m^3/ha. To keep the model more manageable, but without losing any important characteristic related to the hydrological processes, we scaled the aquifer down to 7500 ha which implies that all agents own exactly one hectare. Hence a drop of 2.6666 m per year is caused by a total extraction of 3946.329 m^3/ha times 7500ha. In other words, a decrease in the water table in meters per m^3 of extraction is given by the conversion factor 2,66666m/ 29597474.5m^3 = 0.000000900955461814427m/m^3.

(Martínez-Santos et al. 2008) reports an average rainfall of 435 mm/ha. In term of cubic meters, it results in 10,000m^2 times 0.435 m = 4350 m^3/ha. Given that only a part reaches the aquifer the recharge rate is lower than 4350 m^3/ha. For the numerical application we use a value of 2900 m^3/ha.

Water extraction costs

We calculated the extraction costs as a function of the overall depth of the well (annualized construction and maintenance costs) and of the lifting costs (Tecnoma and Universidad de Cordoba 2004). They are denoted by $c(w,s)$. Based on average values for the area of the western La Mancha aquifer, the cost of wells with a lifetime of 25 years and a depth of 46.83m are 371€ per ha. is 7.94€ per lineal meter of the constructed well depth. Assuming an average price rise of 4% p.a. over 10 years, the nominal costs per lineal meter are 11.75€ per lineal meter. As reported by Llamas and Garrido (2007), it takes 0.004 kWh to draw 1 m^3 of water per meter. At 0.12€ per kWh, water drawing costs per m^3 are 0.00048€ per lineal meter.

Net benefits of agricultural production and water demand

Varela-Ortega et al. (Varela-Ortega et al. 2011), developed a mathematical programming model for four different types of farm characterizing the variety of production systems and farm types found around the western La Mancha aquifer. Using economic, agronomic and policy constraints, the authors calculated the net income for a farm under different water allocation schemes. In other words, they calculated the best response the farmer would make to changes in the amount of water allocated per ha. for the upcoming agricultural campaign, i.e., the changes in inputs other than water

or changes in the type of crop to be planted (to adjust to allocated water changes) to maximize the farm's net income. The best response function with respect to changes in the amount of water is given by the composite input $x^*(w)$. The reported results by (Varela-Ortega et al. 2011), allow us to relate the maximal net income for a farm $\tilde{\pi}$, with respect to optimal use of the composite input x^*, as a function of the water assigned per ha, i.e., $\tilde{\pi}\left(x^*(w)\right)$. For the empirical estimation of this relationship we used their reported data about the optimal farm income and the assigned water and calculated the weighted average of a farm's net income and water consumption for the four different types of farm. Unfortunately, the data that was collected and presented only reflects the upward sloping section of the farm-net-benefit function. Yet, the same authors report that prior to water restrictions being implemented at the beginning of the 21st century, the economic optimum corresponded to extracting 4300 m^3 of water per ha per year. By adding this information as a decrease when consumption surpassed 4300 m^3 in the farm-net-benefit function, we were able to estimate the net benefits of the farm as a function of water consumption. Additionally, we incorporated water extraction costs. The function that best fitted the data is a quadratic function with an R^2 adjusted of 0.94. It is given by

$$\pi(w,s) = 70,3547448 + 0,5061w - 0,0000601w^2 - 7.94s - 11,75s - 0,00048ws$$

$$\underset{(2,36)}{} \quad \underset{(8,94)}{} \quad \underset{(-5,96)}{} \tag{25}$$

In the next step we need to obtain the solution of the optimization problems detailed in equations (21) - (23) where we assumed that the social planner's time horizon is 25 years[15] and the general resource dynamics $\dot{s}$ was replaced by its specifications given in equation (24). Thereafter, based on equation (25) the social planner's problem and the defector's problem were solved numerically in GAMS.

The data obtained from the solution of the optimization models given different initial values of s_0, allowed us to estimate the compliers' and defectors' water demanded that maximizes their net benefits, $w_i^C(s(t),t)$ and $w_i^D(s(t))$, as a function of the available water in the aquifer. Likewise, the optimal water demand for each strategy for different initial values of s_0 allowed us to calculate

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[15] The choice of a planning horizon of 25 years is obviously debatable, but this period of time coincides with the current farming generation leaving the farm to the next and with the economic lifetime of the investment itself.

the farms' optimal net benefits, $\pi^C\left(w_i^C(\cdot),s(t)\right)$ and , $\pi^D\left(\mathrm{w}_i^D(\cdot),s(t)\right)$ as a function of the available water . The results from both estimations are presented in Table O1.

	Defector	Complier
Water demand	$3694{,}3548$ - $3{,}8709$ $s(t)$ (3.72E-9) (2.16E-10) - 1.41E-12 (2.6E-12)	1767.2905 + 112.2303 $s(t)$ (60,4319) (6,7473) -2.0208 $s(t)^2$ + $0{,}0120$ $s(t)^3$ (0,1996) (0,0016)
Farm benefits	$764{,}8887 - 9.8451$ $s(t)$ (39.8792) (0,8659)	$529{,}5613916$ - $0{,}989226044$ $s(t)$ - (13,1712) (2,4326) $0{,}81859727$ $s(t)^2 + 0{,}023769413$ $s(t)^3$ (0,1294) (0,0024) $0{,}000212445$ $s(t)^4$ (1,52909E-05)

Table O1: Water demand and optimal farm net benefits as a function of the available water in the aquifer. Standard errors are presented in brackets.

The social pressure function is specified as a logistic function $\hat{\omega} = \Upsilon \Big/ \left(1 + s_{\max} e^{-(s_{\max}-s)\left(c-\left(c_{ini}-\tau c^2\right)\right)}\right)$,

where $\Upsilon = 2$ indicate the maximal social pressure. The maximal level of the remaining water $s_{\max}$ influences the growth rate of social pressure and the initial share of compliers c_{ini} influences the location of the inflection point of the function.

The numerical application for our game of social networks is based on the estimated values of the parameters presented in Table O1 and the chosen parameter values in Table O2. These values are employed for the generation of the Figures 2 – 6.

Parameter	Value
Number of agents n	7500
Return rate of irrigation water back to the aquifer ψ	0.2

Conversion rate that expresses the change in the depth of the water table as a result of the water extraction μ	9×10^{-8}
Maximal social pressure Υ	0.2
Recharge rate of the aquifer	2900 m^3
s_{max}	80 m
Initial share of compliers c_{ini}	0.4 (Figure 3 $c_{ini} = 0.1$)

Table O2: Summary of the parameter values.

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